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# AROUND AND ACROSS



## Topic

Circumference to Diameter Relationship ( $\pi$ )

## Key Question

How does the distance around the circle (circumference) compare to the distance across a circle (diameter)?

## Learning Goals

Students will:

- generate a graph by using the diameters and circumferences of round objects,
- study the scatter plot to discover the proportional relationship,
- use numeric data from the graph to approximate the value of  $\pi$ , and
- determine a formula for the circumference of a circle.

## Guiding Documents

*Project 2061 Benchmarks*

- *The graphic display of numbers may help to show patterns such as trends, varying rates of change, gaps, or clusters. Such patterns sometimes can be used to make predictions about the phenomena being graphed.*
- *Graphs can show a variety of possible relationships between two variables. As one variable increases uniformly, the other may do one of the following: always keep the same proportion to the first, increase or decrease steadily, increase or decrease faster and faster, get closer and closer to some limiting value, reach some intermediate maximum or minimum, alternately increase and decrease indefinitely, increase and decrease in steps, or do something different from any of these.*

*NRC Standards*

- *Use mathematics in all aspects of scientific inquiry.*
- *Use appropriate tools and techniques to gather, analyze, and interpret data.*
- *Develop descriptions, explanations, predictions, and models using evidence.*

*NCTM Standards 2000\**

- *Use representations to model and interpret physical, social, and mathematical phenomena*
- *Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules*
- *Model and solve contextualized problems using various representations, such as graphs, tables, and equations*

- *Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more complex shapes*

## Math

Measurement

Geometry

$\pi$  ( $\pi$ )

$$2 \pi r = C$$

Proportional reasoning

Statistics

scatter plot

Algebra

slope

equations



## Integrated Processes

Observing

Collecting and organizing data

Interpreting data

Generalizing

## Materials

Metric rulers or meter sticks

Variety of round objects (cans, container lids)

chart or butcher paper.

## Background Information

$\pi$  ( $\pi$ ) is the relationship of the circumference of a circle to its diameter. It is an irrational number. Its approximation is written as  $\frac{22}{7}$  as a fraction or 3.1415... as its decimal equivalent. The approximation of 3.14 is sufficient in all but the most precise applications.

Students are often introduced to the circumference of circles with the formula  $C=2\pi r$ . They do not have any experience for making connections to the formula so it tends to be meaningless and quickly forgotten or mixed up with other formulae.

Using round objects to make a concrete graph provides a valuable experience in which students can think about the relationship of circumference and diameter. They can come to see that the diameter and circumference increase proportionately. They can use the data from the graph to verify this graphically, numerically, and symbolically.

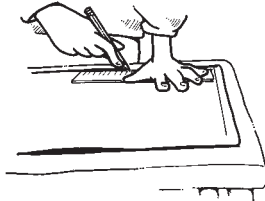
## Management

1. Students need to be familiar with measuring in centimeters to do this activity.
2. This activity can be completed in a 45-60 minute period.

- Before the activity, gather a variety of different- sized round objects such as cans, lids, toy car tires, and other items.

### Procedure

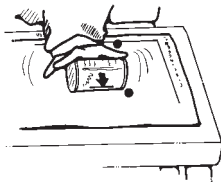
- Discuss the *Key Question* and state the *Learning Goals*.
- Have students construct a graph on a piece of chart paper or butcher paper. Guide them to use a straight edge to draw a horizontal axis just up from the bottom edge of the paper and label it *diameter*. Have them make a vertical axis just in from the left edge of the paper and label it *circumference*. (The intersection of the two axes in the lower left corner of the paper is the origin.)



- Inform the students that for each round object, they must find the point on the horizontal axis that corresponds with that object's diameter. Show them that they can do this by putting the diameter of the round object on the horizontal axis with one end on the origin. Have them mark the other endpoint along the axis to show the distance of the diameter from the origin.

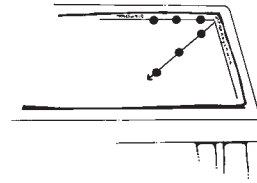


- Instruct the students to make a small line on the circumference of the round object. Have them place this line on top of the dot they placed on the horizontal axis which indicated the object's diameter. Inform them that the object must be positioned so that it can be rolled up the paper, parallel to the vertical axis.



- Instruct the students to roll the round object one complete revolution until the mark is back on the paper. Have them mark a point at this position on the paper. (This point represents the relationship between the circumference and diameter of the can.) Caution students to keep the round object rolling parallel to the vertical axis.

- Have the students graph the points for all of the round objects.



- Discuss what patterns can be seen in the graph and what information it gives about round objects. [straight line, goes up as it goes right; circumference is always bigger than diameter, bigger diameters result in bigger circumferences]
- Have the students measure and record the diameters and circumferences of the round objects using the graph.
- Direct them to calculate and record the ratio of circumference to diameter for each of the round objects, and then average the ratios.
- Discuss how the ratios compare to each other, what that says about circles, and how it is related to the graph.
- Have the students write an equation to summarize what they have learned about circles.

### Discussion

- What patterns do you see in the graph? [straight line, goes up as it goes right]
- What do the patterns tell you about circles? [circumference is always bigger than diameter, bigger diameters result in bigger circumferences, circumference grows proportionally to diameter]
- What is the average ratio of circumference to diameter for all the objects? [should approach pi, 3.14]
- What does this average tell you about circles? [how many times bigger the circumference is than the diameter]
- How is the average related to the graph? [tells how much the lines go up for each step to the right, the slope]
- How could you determine the circumference of a circle if you knew the diameter? [use the graph by interpolating or extrapolating, multiply the diameter by the average]
- Write an equation you can use to determine the circumference of a circle if you know the diameter. [ $\pi (d) = C$ ]

### Extension

Measure the diameters of large circles which have been painted on athletic fields and predict their circumferences. Then measure them to see if they are correct.

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## Key Question

How does the distance around the circle (circumference) compare to the distance across a circle (diameter)?

## Learning Goals

### ***Students will:***

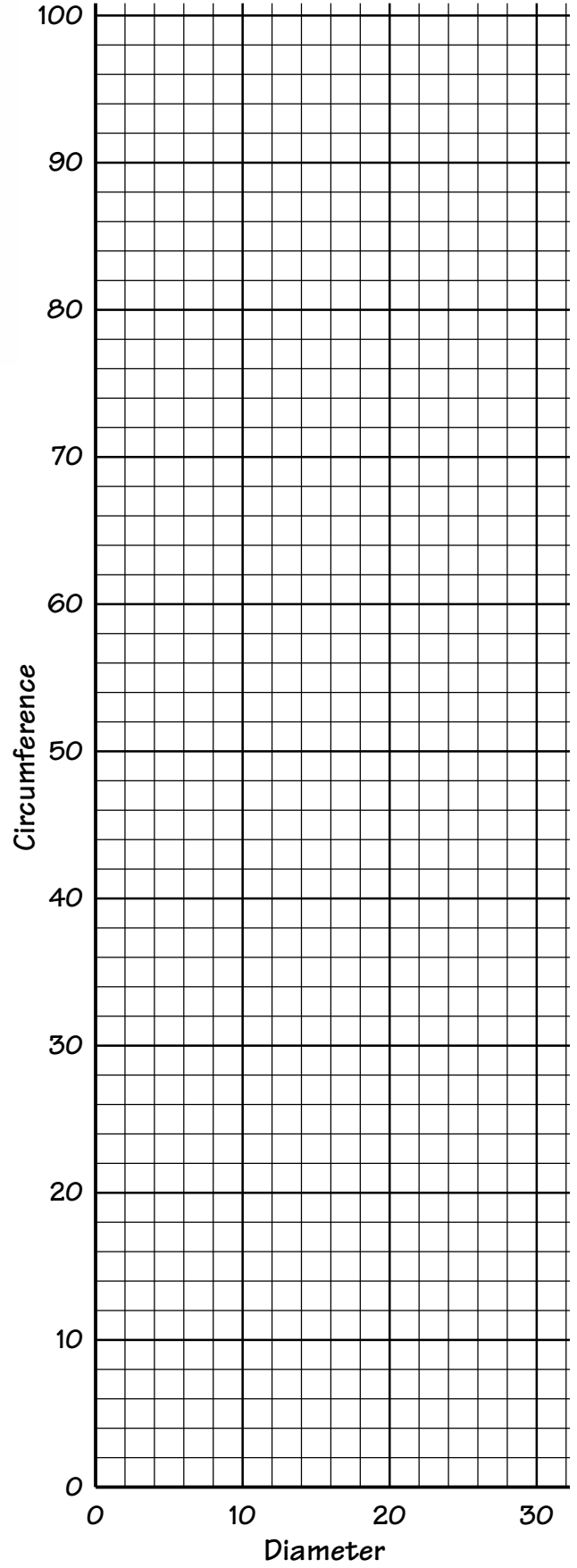
- generate a graph by using the diameters and circumferences of round objects,
- study the scatter plot to discover the proportional relationship,
- use numeric data from the graph to approximate the value of pi, and
- determine a formula for the circumference of a circle.



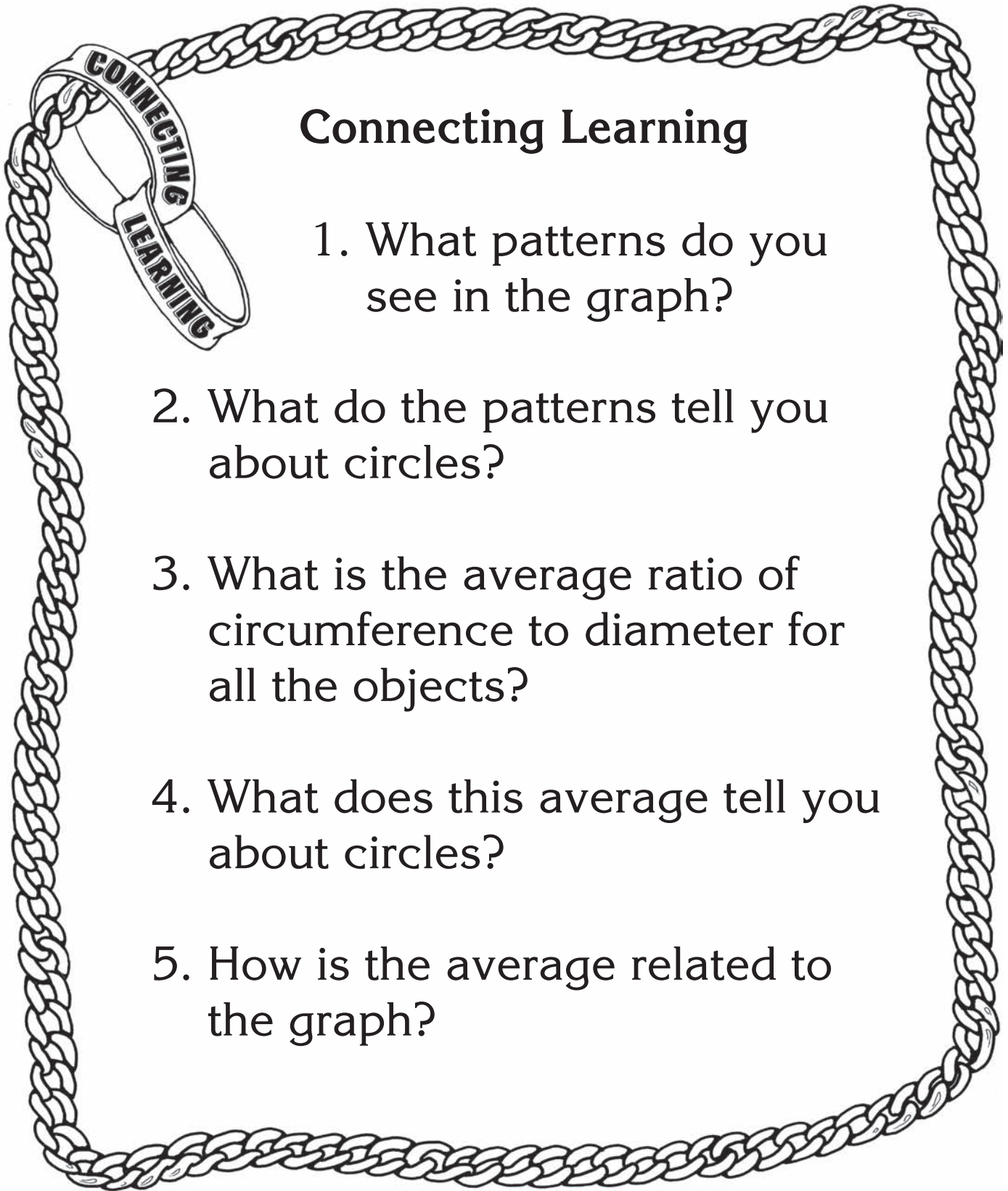


How does the circumference of a circle compare to the diameter of a circle?

Round Object	Diameter	Circumference	Ratio $\frac{C}{d}$
Average			



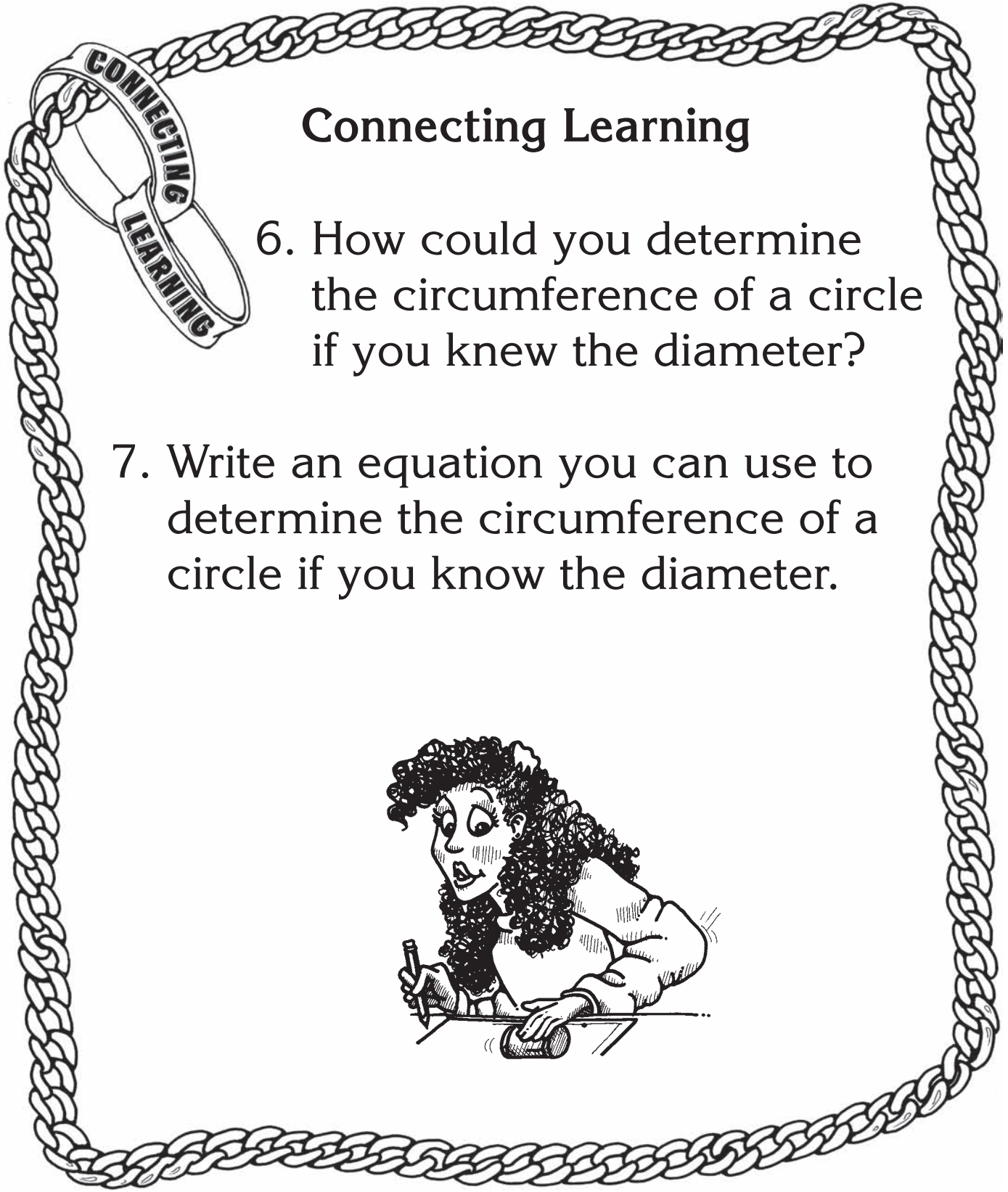
# AROUND AND ACROSS



## Connecting Learning

1. What patterns do you see in the graph?
2. What do the patterns tell you about circles?
3. What is the average ratio of circumference to diameter for all the objects?
4. What does this average tell you about circles?
5. How is the average related to the graph?

# AROUND AND ACROSS



## Connecting Learning

6. How could you determine the circumference of a circle if you knew the diameter?
7. Write an equation you can use to determine the circumference of a circle if you know the diameter.

