Maximizing Math
Playing with Palindromes
by Dave Youngs

This month's *Maximizing Math* activity deals with palindromes. Palindromes are words, phrases, or numbers that read the same from left to right and right to left. The *Puzzle Corner* activity elsewhere in this issue is a good introduction to palindromes for students who are not familiar with them, and you may want to have students do that activity before beginning this one.

The word palindrome comes from the Greek word *palindromos* which means running back again. The early Greeks were the first to play with palindromic words and phrases over 2000 years ago and this fascination has not abated in the intervening years. (The Internet is full of sites dedicated to sharing palindromes in multiple languages.) It is my hope that this activity will help your students get a sense of this fascination as they deal with a special set of palindromic numbers. In the process, they will do lots of arithmetic and mathematics while they explore the patterns and symmetry of palindromes.

In *Playing with Palindromes*, students look at what happens when certain palindromic numbers are doubled and squared. (For students who are not familiar with squaring, this activity is a good introduction to the concept.) In the first part of the activity, students are asked to determine the smallest counting number palindromes (the counting numbers are 1, 2, 3, 4, ...) having one through four digits. These palindromes are 1, 11, 101, and 1001. They are asked to write these numbers in the table provided and then to double and square each of these original palindromes and record the results in the table. Interestingly, the doubles (2, 22, 202, 2002) and the squares (1, 121, 10201, 1002001) are also palindromes. (What would happen if these doubles and squares are doubled and squared?) Students are then challenged to determine the smallest seven-digit palindromic number, its double, and its square (1000001, 2000002, and 1000002000001) by applying the patterns they discovered while completing the table.

Part Two of the activity continues this exploration with a different set of palindromes consisting entirely of ones (1, 11, 111, 1111, 11111, etc.). As in the first part, students are asked to double (2, 22, 222, 2222, etc.) and square (1, 121, 12321, 1234321, etc.) these palindromes. As students begin these calculations with the first few palindromes in this series, they will discover patterns that can be used to extend the doubles and squares without having to do the computations. In this instance, the doubling pattern will continue infinitely, but the squaring pattern only continues up to 111111111 (nine ones) whose square is 12345678987654321. The last part of the activity asks students to predict what will happen when 1111111111 (ten ones) is doubled and squared and then to do the calculations using the template provided. When they do, they will discover that the pattern for squaring no longer holds since the square is 1234567900987654321. Lastly, students are challenged to think of some other palindromic numbers and to play around with them. If they discover anything of interest, they should be prepared to share it with other students in the class.

I hope you find this activity worthwhile. There'll be another one in the next issue. If you have any questions or comments, please feel free to contact me. My email is dyoungs@fresno.edu.

Playing With PALINDROMES

Doubling and Squaring

A palindromic number is one that is the same when read from left to right and right to left. The year 2002 is an example of a palindromic number. When you double this number you get 4004, which is also a palindrome. When you square 2002 (multiply it by itself) you get 4008004—another palindrome. Not all palindromic numbers produce palindromes when doubled or squared, but some do. In this activity you will explore a few of these special numbers.

What is the smallest palindromic counting number? (The counting numbers are the infinite set of numbers that begin with 1, 2, 3, 4, 5, ...) What is the smallest two-digit palindromic counting number? ...the smallest three-digit palindromic counting number? (Don't let this one trick you.) ...the smallest four-digit one? List these palindromes in the first row of the chart below. Next, double each one and put the resulting numbers in the second row. Finally, square each of the original palindromes and record the answers in the third row of the chart.

Counting Number Palindromes

	Smallest one-digit palindrome	Smallest two-digit palindrome	Smallest three-digit palindrome	Smallest four-digit palindrome
Original palindrome				
Original palindrome doubled				
Original palindrome squared				

What patterns do you notice in the above numbers? List as many patterns as you can here, then use them to find the smallest seven-digit palindrome, its double, and its square.

Playing With PALINDROMES

Part Two

1. Here is another series of palindromes. Find the double and square of each and record it in the table below.

Palindrome	1	11	111	1111
Doubled				
Squared				

2. What patterns do you see in the table?

3. Use these patterns to fill in the doubles and squares for the next five numbers in the series.

Palindrome	Doubled	Squared
11111		
111111		
1111111		
11111111		
111111111		

4. Predict what will happen to these patterns with the next palindrome in the series—1111111111.

Playing With PALINDROMES

Part Two

5. Use the spaces provided to test your predictions.

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  1 1 1 1 1 1 1 1
+ 1 1 1 1 1 1 1 1
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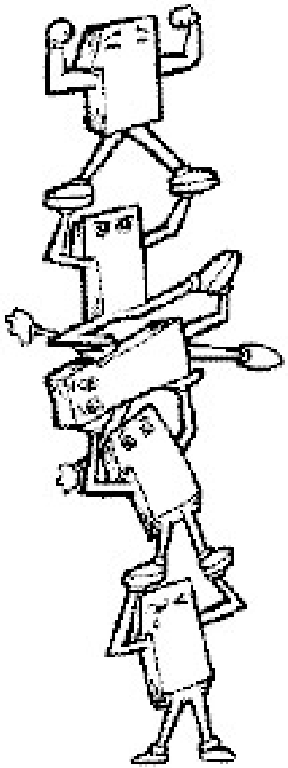

										1	1	1	1	1	1	1	1	1	1
x										1	1	1	1	1	1	1	1	1	1
																			0
																		0	0
																	0	0	0
																0	0	0	0
															0	0	0	0	0
														0	0	0	0	0	0
													0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0
+											0	0	0	0	0	0	0	0	0

6. Explain why one of the patterns does not hold true for the 10-digit palindrome.

7. Think of another series of palindromic numbers. Play around with these palindromes and see what you discover. Be ready to report your findings to the class.