Puzzle Tiles

Puzzle Corner

by Dave Youngs

Introduction

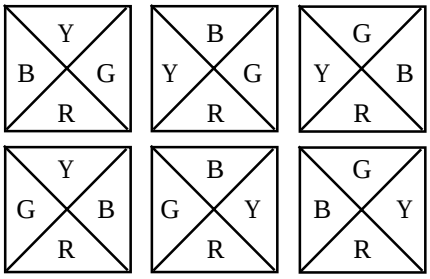
This month, the *Puzzle Corner* activity is designed to be done following the *Maximing Math* activity in which students discover the coloring scheme for the six tiles used in this puzzle. If your students have not done *Multi-colored Tiles*, they need to do so before going on.

I am indebted to Dr. Richard Thiessen for his help in developing these activities. As far as we know, this puzzle has not been done before.

This Month's Puzzle

Puzzle Tiles challenges students to make several configurations using six square tiles, each of which has been divided into four regions and colored to be unique. The first sheet has twelve tiles that can copied onto cardstock or some other similar material and then cut apart to provide tiles for two students. To facilitate cutting them out, they are drawn touching. It would be helpful to have envelopes available in which students could store their tiles when they are not working on the puzzles.

After the students each have six tiles, they will need to color them according to the guidelines in the *Maximizing Math* activity. It is important that each tile is colored correctly or the puzzle will not work. Use the following key if you aren't sure your students have colored the six tiles correctly. Note that one color, red in this case, has been placed at the bottom of each tile. With this done, it is easier to find the unique ways to add the other colors around the tile. Also note that there are three pairs of mirror images in the set of tiles. Discovering these pairs and using them in the puzzles is an important part of this activity.

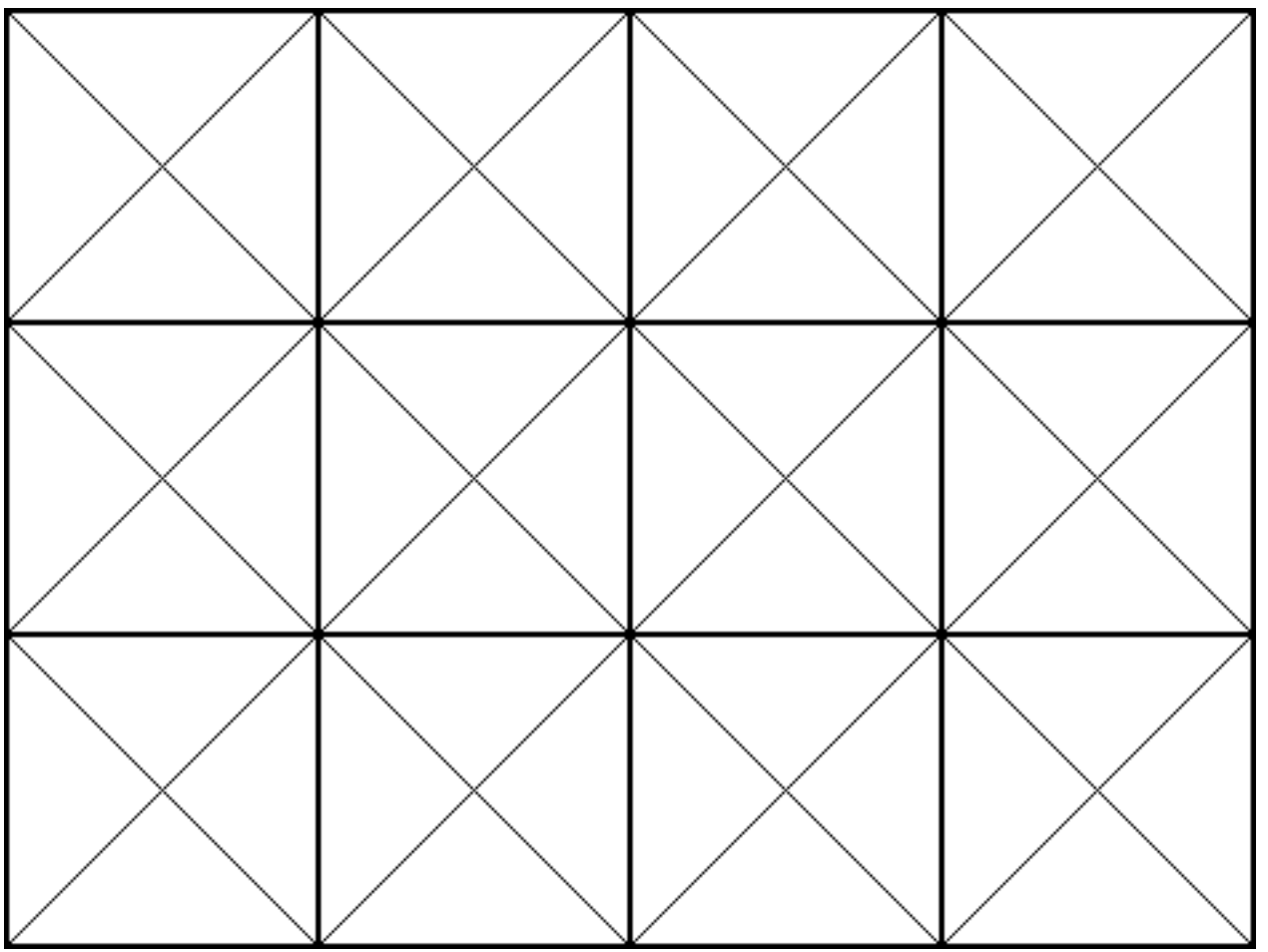


Once all students have correctly colored sets of tiles, they are ready to work on the challenges presented on the second sheet. Each challenge is accompanied by an area where solutions can be recorded. Some of the challenges are harder than others, but all are possible.

In the *Puzzle Corner* tradition, solutions will not appear until next month's column. I hope that you and your students find this puzzle interesting and worthwhile. I'll have another one for you next month.

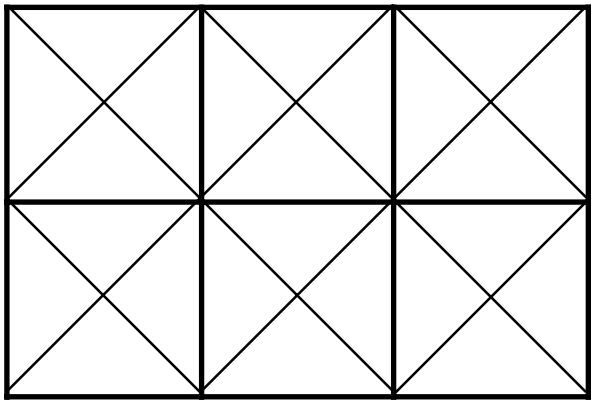
Puzzle Tiles

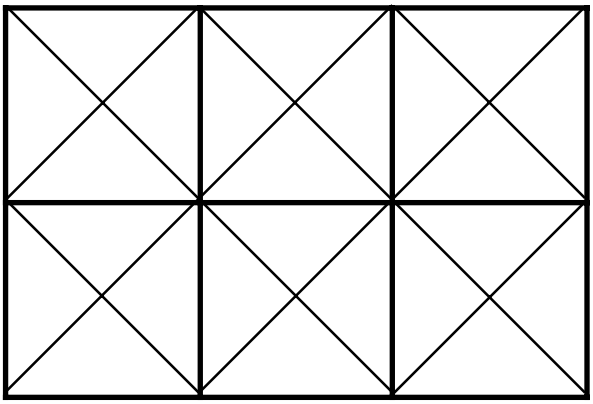
Cut out six of the square tiles below and color them using the patterns you discovered in the *Maximizing Math* activity.



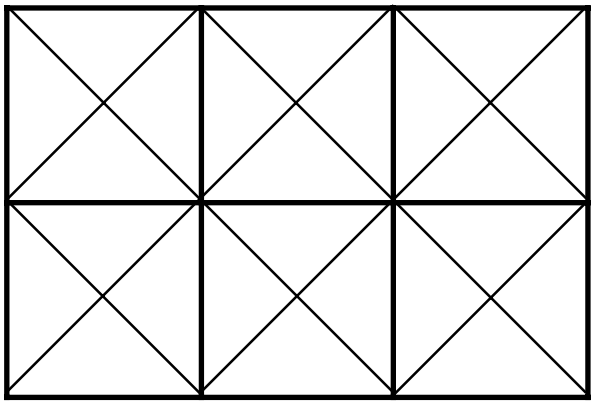
Puzzle Tiles

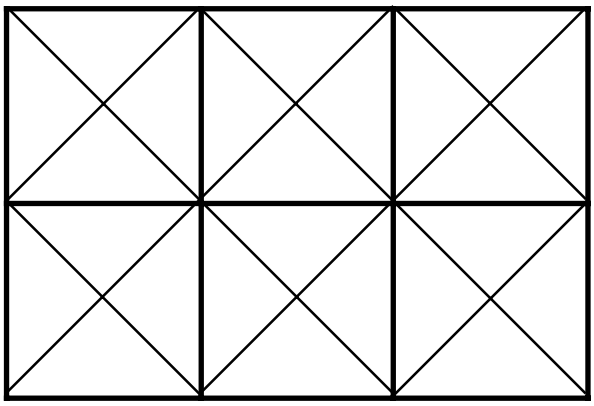
Challenge #1:
Place the six tiles in a rectangle so that no two touching sides have the same color. Do this in two different ways and record your solutions below.



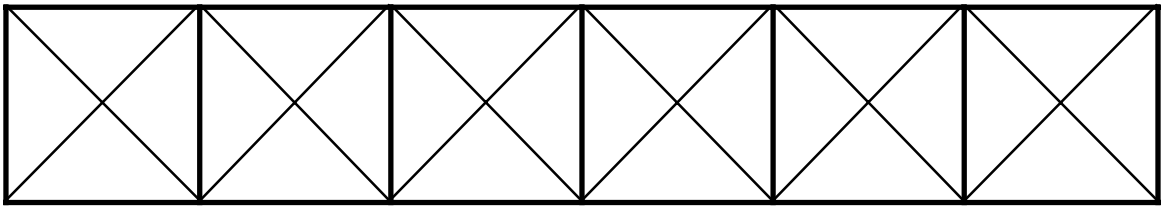


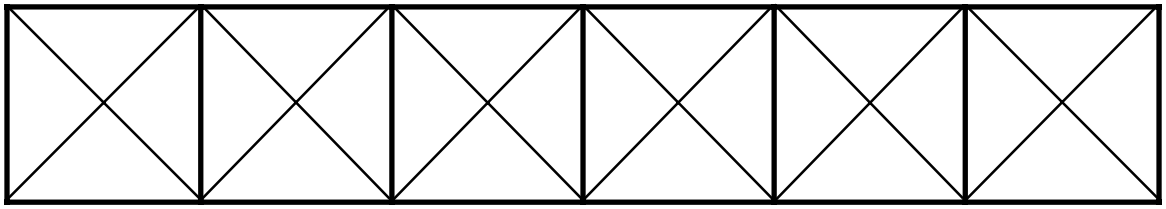
Challenge #2:
Place the six tiles in a rectangle so that wherever two sides touch, they have the same color. Do this in two different ways and record your solutions below. What patterns do you notice in this arrangement?



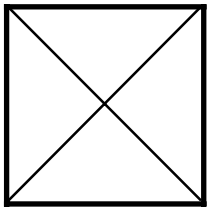


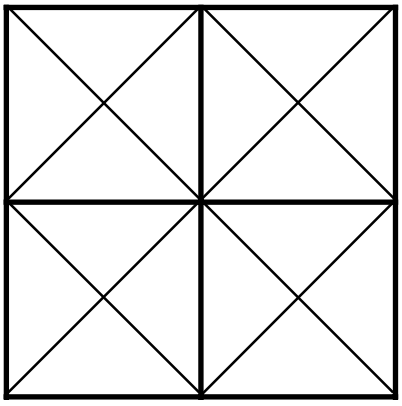
Challenge #3:
Place the six tiles in a row so that the bottom triangle in each one is the same color and the triangles on the left and right sides match to produce diamonds. Is there a mirror image in your row? [To find out, hold a mirror between the third and fourth tiles and see if what the mirror shows is the same as the original configuration.] If you haven't made a mirror image, try again until you get one. Record your solutions below.

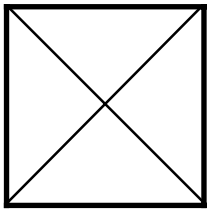




Challenge #4:
Pick one of your tiles at random. Record it in the first box on the left. Use four of the remaining tiles to make a square that has four interior diamonds that match the colors, and order, of the tile on the left. Record your solution and then color in the tile that is left over in the box to the right. Study your solution and see what you discover. Pick another tile and repeat this process. Make a record in the second set of figures below. How did what you learned the first time help you the second time?







Challenge #5:
Make your own puzzle challenge.

Multi-colored Tiles

Maximizing Math

by Dave Youngs

Introduction

This month, the *Puzzle Corner* and *Maximizing Math* activities are designed to be done together. *Multi-colored Tiles* should be done first because students will discover the color schemes needed for the six square tiles used in the *Puzzle Corner* activity. I am indebted to Dr. Richard Thiessen for his help in developing this activity.

This Month's Activity

In *Multi-colored Tiles* students are shown various-shaped tiles divided into different numbers of regions. They are asked to find all the unique ways to color each shape of tile following three rules.

1. The number of colors to be used for each tile must correspond to the number of regions into which it is divided. For example, the square tiles have four regions, so four colors are used for each square.
2. Each of the colors used must appear on each tile. This means that the triangle, for example, which is divided into three regions, must have each region colored with a different color.
3. Each tile of a given shape must be colored in such a way that it is unique, but still follows the above rules. This means that each tile must be different from the others of the same shape, even after it is rotated.

The first tiles on the sheet are circular ones, which are divided into two regions. There is only **one** unique solution to make these tiles using two colors. It is important that students discover this and understand why before going on to the next part of the activity. It may be necessary to have some students cut out the colored circles and rotate them before they really understand that a circle with red on the right half and blue on the left half is the same as one with blue on the right and red on the left–when it is rotated.

The triangular tiles, which appear next on the sheet, can be colored in only **two** unique ways using three colors. Again, some students may not be able to see this unless they cut the triangles out and rotate them.

The square tiles, each of which is divided into four regions, appear next. These tiles will be used in the *Puzzle Corner* activity that follows this one, so care should be taken to find all **six** ways these tiles can be made using four colors.

Extensions

After students have successfully colored their tiles, they are challenged to come up with an interesting extension to explore. There are many extensions possible. An obvious one would be to find out how many ways pentagons divided into five regions could be colored (there are 24 ways to do this). Another extension might be to find out in how many ways the tiles could be colored, and still be unique, if rotations were not allowed (two ways for the circles, six for the triangles, 24 for the squares). Another extension might be to change the requirement that the number of colors used corresponds to the number of regions. For example, instead of requiring four colors for the square tiles, you could allow five or six, or even two or three colors. A further extension would be to change the requirement that each region of a tile be colored a different color. In this way the triangular tile could be colored all one color or have two regions colored one color and the third region a different color, in addition to having each region colored a different color.

An optional page has been included for the pentagon extension. The pentagons are divided into five regions. The challenge of finding all the unique ways (24) they can be colored using five colors is a rather laborious task, and one that needs to be done systematically in order to insure that all the unique tiles are found. (Since the tiles can be rotated, one method is to use the same color for the bottom region every time; another method is to use numbers or letters to find all the possibilities and then use this code to color the tiles.) Once this is done, older students might be asked to predict how many different ways hexagons with six regions could be colored with six different colors. (There are 120 ways.) A t-table could then be made of this information and students challenged to come up with a generalization for an n-sectioned figure with n-colors [(n-1)!].

Teaching Suggestions

This activity may take a fair amount of teacher facilitation, even though it was designed so that students start with easier tasks before proceeding to harder ones. It is important that students understand the rules for coloring the tiles so that unique tiles are produced. As stated previously, some students may need to cut out the tiles before they "see" that two tiles that appear to be colored differently are actually the same when rotated.

Students may need some help with thinking of extensions to explore. It may be helpful, at this point, to have a class session to brainstorm some extensions.

As in all *Maximizing Math* activities, communicating mathematically is a critical part of the process. The teacher needs to talk with students as they work on the activity and encourage students to talk to each other about the activity. In addition, enough time needs to be set aside at the end of the period for students to engage in a class discussion on the things they have discovered in doing this activity.

The square tiles produced in this activity are also used in the *Puzzle Corner*; it would be appropriate to follow this activity with the *Puzzle Corner*.

Background Information

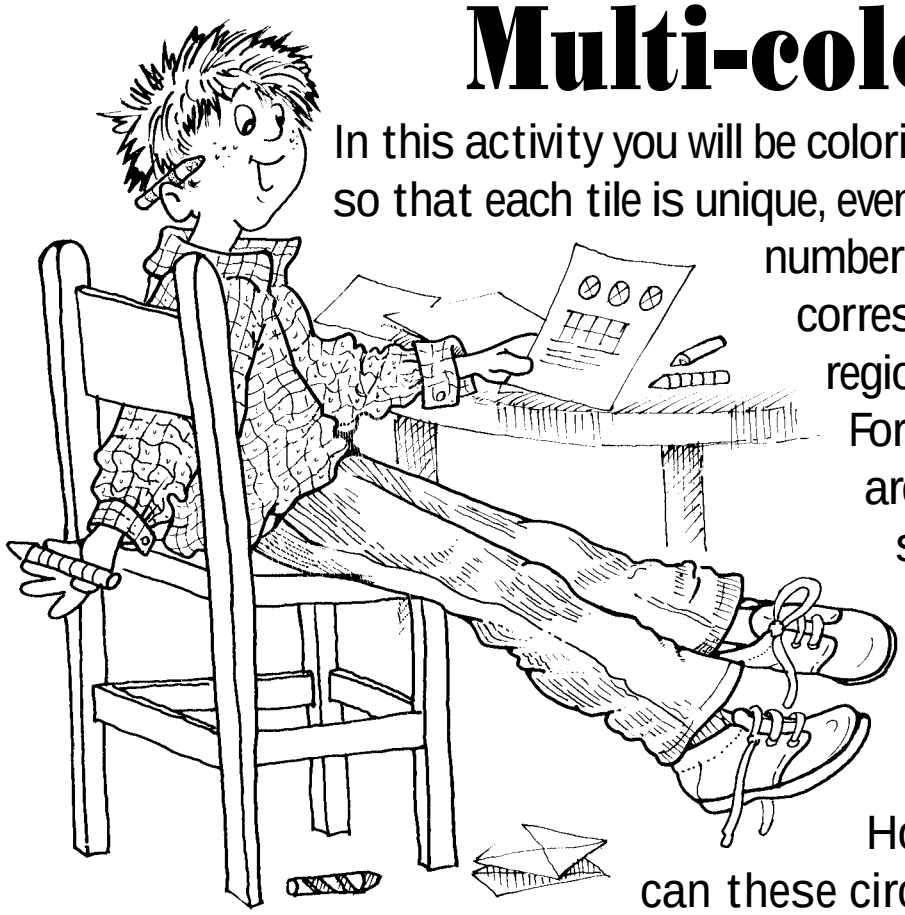
There is a wealth of mathematics in these tiles. Patterns play a key part as does spatial visualization. Mirror images occur with all of the tiles and will be explored in more depth in the *Puzzle Corner* activity. The fundamental ideas of counting, permutations, and good order also play a part here and can be addressed with older students. Older students can also get into some algebra and may be challenged to come up with generalizations for coloring n-sided polygons divided into n-regions.

Summary

I hope this activity proves to be valuable for your students. I'll have another one for you next month.

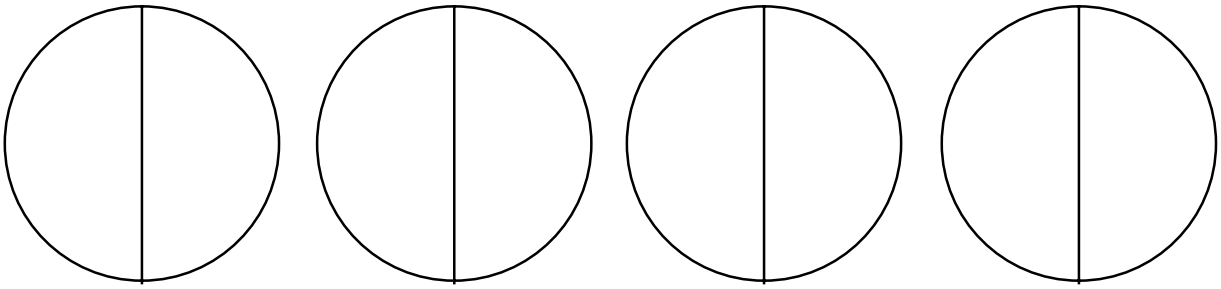
I would appreciate any feedback you can give me on this, or any other *Maximizing Math* activity. You can send mail to Dave Youngs, AIMS Education Foundation, P.O. Box 8120, Fresno, CA 93747 or e-mail to dyoungs@fresno.edu.

Multi-colored Tiles

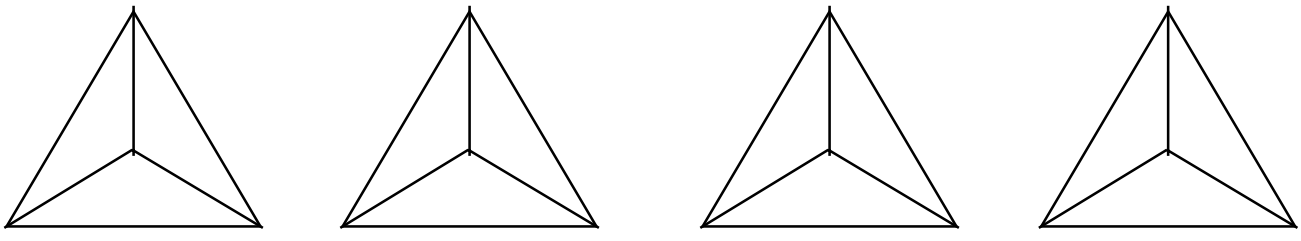


In this activity you will be coloring different-shaped tiles so that each tile is unique, even after it is rotated. The number of colors you use must correspond to the number of regions the tile is divided into. For example, the square tiles are divided into four regions so four colors should be used. Each tile must have each of its regions colored a different color.

How many different ways can these circular tiles be colored? Remember, you can only use two colors and each tile must be unique, even after being rotated. Show your solution(s) and justify your answer.



How many ways can the triangular tiles be colored using three colors? Show your solution(s) and justify your answer.



How many unique ways can the square tiles be colored using four colors? Show your solution(s) and justify your answer.

Challenge: Think of some extensions for this activity. List a few of them. Pick one extension to explore further and show what you find out.

Pentagon Challenge