The Vee Diagram: A Guide for Problem Solving

by Richard Thiessen

In his article, Puzzles *as Problem Solving Tools* (*Newsletter*, March 1992), Dave Youngs makes a point of the importance of having students analyze the process or processes used in solving a puzzle problem. He states "...students should be analyzing the solution, checking its validity, looking for patterns and relationships suggested by the solution, and bringing closure to the problem by discussing the mathematical processes, concepts, and relationships involved in finding the solution."

How can we help students to be more reflective about their problem solving activities? Or perhaps, more importantly, how can we be more reflective about the problem solving activities we pose for our students? How can we better facilitate and assess the thinking of our students in problem solving situations? D. B. Gowin, Professor of Biology at Cornell University, asked similar questions about what was happening in the lab experiences of his students. While they were able to complete their experiments, it was unclear how much students understood of what had taken place in the experiment or how what they had done related to the processes, concepts, and relationships that they were presumably learning in the lecture portion of the course. To provide some structure within which his students could better think about what was happening in the lab situation, he developed the idea of the *Vee diagram.* I believe the Vee diagram can serve as a way to structure and guide our thinking and that of our students in a variety of mathematics learning situations.

Gowin's concerns in connection with his student's lab experiments are not unlike the concerns that many of us have as we attempt to guide our students in problem-solving activities, in using manipulatives, and in doing AIMS activities. Teachers often say that while students may use manipulatives, they fail to make the connection between the manipulative and the concept that is presumably being embodied by it. Students use the manipulative to guide their thinking for a time, but after they go back to recording their work with paper and pencil, their thinking seems to go back to what they have learned by rote. Could it be possible for students to engage in an AIMS activity, to find it interesting, and yet fail to relate the activity to the concepts and relationships involved?

In this article I want to describe Gowin's Vee diagram and suggest some ways in which it can be used as a tool to guide our thinking and that of our students in problem-solving situations. While we will be focusing on its use in problem solving, it will be clear, as has already been suggested, that there are other applications of this tool in the teaching/learning process.

Elements of the Vee

Figure 1 shows a Vee diagram with each of its elements labeled. At the top is a *focus question* which expresses what we want to find out. The focus question is directed to an *event* and the *objects* used in the event. The left side of the Vee is *conceptual.* Here are listed the *conceptualstructures, relationships*, and *concepts* that are or might be involved in finding an answer to the focus question. The right side of the Vee begins with making *records* of the event. The *transformation* and *interpretation* of these records may involve reorganization and rearrangement of the records. It may involve constructing tables, charts, and graphs from which conclusions can more readily be drawn. The *knowledge claimsare* the answers to the focus questions. *Theseclaims should be the products of interaction between the conceptual and the methodological elements of the Vee. The relation ofconcepts to the event is essential.* Finally, there is a *value* claim. What is the worth of the knowledge claim or of the process that leads to that claim?

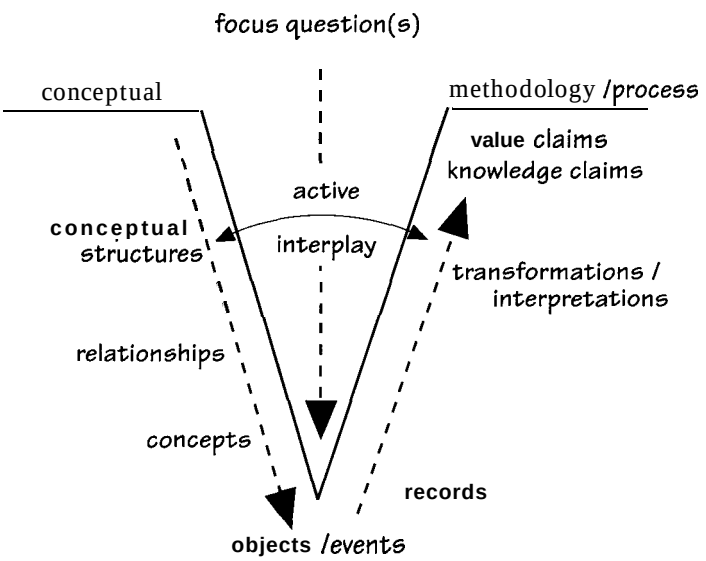

Figure 1

The remainder of the article will be devoted to using two puzzle problems to exemplify the Vee diagram. While it can be used by both the teacher and students, our focus is on its use as a tool to help us, as teachers, organize and guide our thinking about not only the process of solving the puzzle, but also-and more importantly-the knowledge being constructed by the problem solver as the result of an interplay between the related concepts and the puzzle-solving activity. It should be clear that much more is going on than simply putting some pieces together to form a particular shape.

Rectangle Puzzle

This puzzle consists of six pieces (*Figure 2*) that may be made of card stock, wood, or plastic. The problem is to construct a rectangle using all six of the pieces.

Figure 2

This is a typical puzzle problem. It provides the problem solver with a set of objects and poses a problem to be solved by doing something with the objects (event). It is a problem that middle school students find interesting and one they can readily solve. Using trial and error they can put the pieces together to form a rectangle. While solving the puzzle in this way is a valuable experience for the student, there is much more that can be learned from the experience. The Vee diagram provides a way for both teacher and student to think about the "much more" that can be gained from this experience. *Figure 3* shows one way to construct a Vee diagram for the rectangle puzzle.

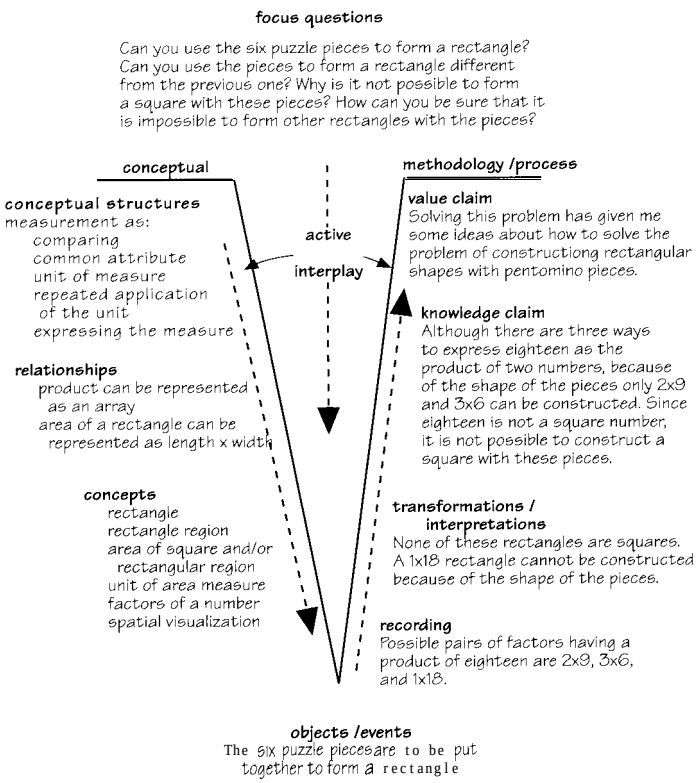

Figure 3

We begin with the *focus questions*:

* Can you use the six puzzle pieces to form a rectangle?
* Can you use the pieces to form a different rectangle from the previous one?
* Why is it not possible to form a square with these, pieces?
* How can you be sure that it is impossible to form any other rectangles with the six pieces?

These are directed at the *object/event* which is the set of puzzle pieces being used to form rectangles. The first question may well result in the student simply using trial and error to find a way to construct a rectangle. The second question, however, should cause the student to begin to consider whether a second rectangle is possible. Hopefully, this consideration will be based on the fact that, using one of the square pieces as a unit, the area of any rectangle constructed with these pieces will have an area of eighteen square units. This can lead the student to consider (1) the possible products of eighteen, and (2) a recognition of the connection between multiplicaition represented as an array and the area formula for a rectangular region.

The student may make a record of the possible rectangles having an area of eighteen by listing all possible pairs of factors having a product of eighteen. This should also lead the student to notice that a 3 x 6 rectangle is the same as a 6 x 3, and so only one of these would be listed; this is, of course, due to the commutative property of multiplication. Finally, answering the question about constructing a square shape should cause the student to recognize that since eighteen is not a square number, it is not possible, using these pieces, to construct a square shape having an area of eighteen.

It is important to pose focus questions that lead to the sort of interaction between the conceptual and the method / process sides of the Vee that we have just described. This interaction should be further reflected in the student's knowledge claim.

Since this Vee diagram was designed to guide the teacher's thinking about this problem, the value claim was related to how this puzzle problem might enhance the student's understanding of certain concepts and relationships. If the Vee diagram had been constructed by a student as a way to communicate her thinking about the problem to a teacher, the value claim might be something like, "solving this problem has given me some ideas about how to solve the problem of constructing rectangles with pentomino pieces." What is claimed as a value of a particular problem will depend on the purpose for which the Vee diagram is being used.

The knowledge claim that results from answering the focus questions for this six-piece puzzle might well suggest related questions and problems. For example, suppose we asked the same focus questions, but changed the puzzle to one having eight of the T-shaped pieces and four of the squares. Focus questions and the left side of the Vee diagram might stay the same; however, the object / event would be modified and the right side would change somewhat because of the additional pieces.

The rectangle puzzle which appears in the ***Puzzle Corner*** of this *Newsletter* has a layout of the six pieces that may be duplicated. Make several copies of these pieces, and try your hand at solving the modified puzzle. Monitor your thinking by using the Vee diagram. Can you express your answers to the focus questions as a knowledge claim? Are there other questions that occur to you as the result of this activity? Can you make a value claim for this modified activity?

Conway's Cube

I first encountered this puzzle about a year ago at the National Council of Teachers of Mathematics convention in Nashville. The National Science Foundation booth had a series of puzzle problems on display and this was one of them. When I got back and was describing the puzzle to another faculty member, she remembered that it had been written up in the ***Oregon Mathematics Teacher.*** A few days later, I was showing the puzzle to Jim Wilson, a regular contributor to the *Newsletter*, and he told me that it is called ***Conway's Cube,*** named after the English mathematician, John Conway. After so many encounters with this puzzle in such short order, I concluded that it must be a good one.

The puzzle involves six 1 x 2 x 2 pieces, three cubes (1 x 1 x 1), and an open 3 x 3 x 3 box. The six 1 x 2 x 2 pieces can be constructed by gluing cubes together, and the box constructed with tagboard. The problem is to pack the six pieces into the box.

Figure 4

Let's examine how we might use the Vee diagram to think about the process of solving this puzzle problem. Clearly, the *object/event* is to pack the pieces into the box. So we know what to put at the bottom of the Vee. What are some of the concepts and relationships that are involved in this puzzle? I have listed some that occur to me ***(Figure 5).***

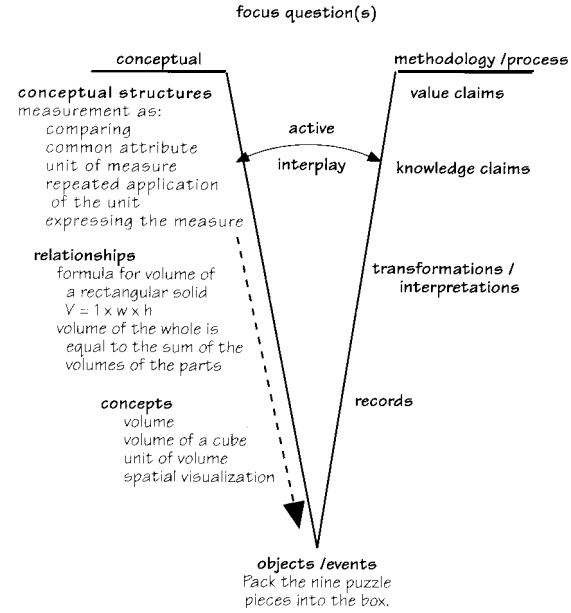

Figure 5

Knowing the object/event and some of the concepts and relationships that are involved, we are ready to formulate the focus questions. These questions should relate the concepts and relationships to the puzzle and the answers should be reflected in the knowledge claim. So what are some questions we might ask? I have suggested several (*Figure 6*).

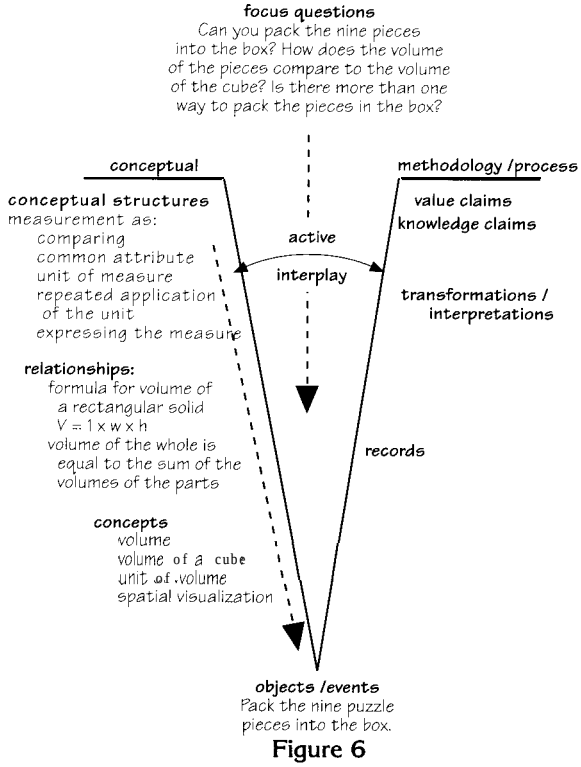

Figure 6

I would expect that the order in which the student answers them might vary. For example, if the student was

(Please see VEE, page 11)

successful in answering the first question fairly quickly, then the answer to the second question would involve reflecting on what was done. However, if the student has difficulty packing the pieces, then the second question provides a way to determine that since the total volume of the pieces is the same as the volume of the box, there is at least some hope that they might fit. The third and fourth questions can only be answered after successfully solving the puzzle.

The knowledge and value claims (*Figure* 7) are some that were recorded by students who have worked on this problem with me. While the knowledge claims may not express any significant generalizations about packing problems, they do reflect that the students are doing more than just packing the pieces into the box: they are relating the relevant concepts and relationships to the puzzle activity (objects / event).

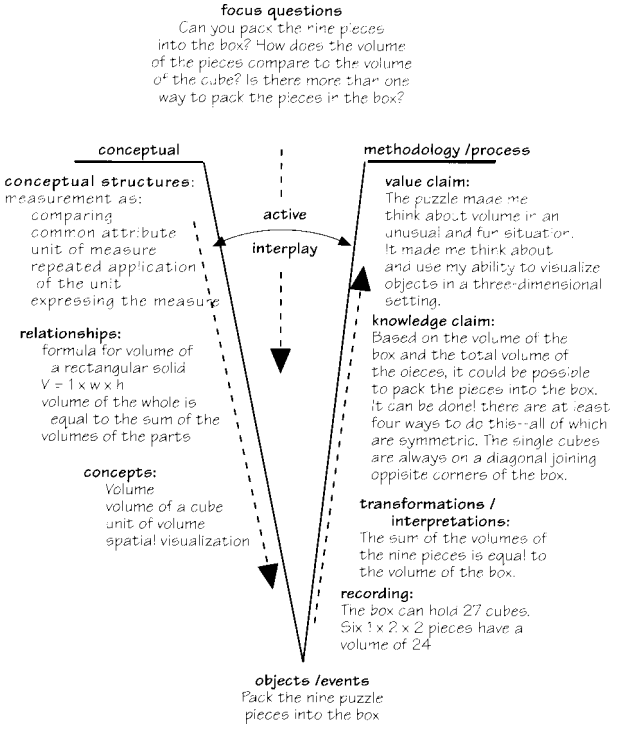

Figure 7

Benefits of using the Vee

After using the Vee recently with a group of teachers, one of them said excitedly, " I'm going to use this with my problems of the week. It will help me think about what I want my students to do with the problems and how I want them to express their solutions. It also helps me think about the concepts that are involved and how the problem of the week relates to other things I'm doing or have done with my students."

Our belief that students must construct their own knowledge leads us to find ways to provide them with a variety of hands-on activities. Assuring that students relate these activities to the relevant concepts and relationships is our challenge. I believe the Vee diagram is a tool that can help us meet that challenge. You can expect to see more applications of the Vee in future articles.