



Maximizing Math

Junk's Puzzle: A Mathematical Look

by Dave Youngs

This *Maximizing Math* activity explores the mathematics in the third puzzle in this month's *Puzzle Corner* column. Students should be proficient with that puzzle before doing this activity. As mentioned in the *Puzzle Corner*, this puzzle is a modification of a sliding block puzzle created by Toshi Junk Kato, a puzzle inventor active in the international puzzle community. Junk's original puzzle—which is the ultimate challenge presented at the end of the *Puzzle Corner*—is a wonderful, but complex, mathematical environment. Its 161 moves and its $2(3^{n-2}) - 1$ generalization (where n is the number of blocks) are too advanced for most elementary students, but are appropriate for older students.

The modified version of Junk's puzzle presented here is also quite mathematical in nature, but a little less intimidating than the original. Solving it takes 1, 3, 9, 27, or 81 moves, respectively, for one to five blocks. This generalizes to 3^{n-1} moves, where n is the number of blocks. Fortunately, this relationship to the powers of three is easily seen once students have mastered the puzzle. In addition, several other mathematical patterns become apparent as students work on the puzzle.

To do this activity students need to use the frame for Puzzle 3 on the last page of the *Puzzle Corner* and all five blocks. The *Maximizing Math* activity then takes students through a systematic exploration of the mathematics in the puzzle.

As students work through this activity and try to determine the number of moves it takes to switch all five blocks, they will employ a powerful problem-solving technique: doing a simpler problem and then applying what they learn to solve the more difficult problem. Mathematicians often employ this strategy when they encounter a difficult problem.

As students follow the directions on the student sheet, they use this problem-solving strategy. They start by moving only one block, then two, and so on until they have moved all five blocks. As students fill in the tables, they should discover that the number of moves for each subsequent block is three times greater than the previous number of moves. As they record the moves for the first three blocks (1, 3, 9) for example, they may be able to predict that adding the next block will take 27 moves. If not, they can do the puzzle and discover this. When they have the sequence 1, 3, 9, 27 they should be able to predict that switching five blocks will take a total of 81 moves.

Once students have seen the pattern in the total numbers of moves, they may be ready to track the number of moves each block makes. Doing this requires very accurate record keeping and is best done in groups. One student who is very comfortable solving the puzzle moves the pieces while other students tally the moves by individual blocks. For example, one student could count only the number of moves made by block A, another student could keep track of block B, etc. To reiterate, it is important that the data are accurate or the patterns won't be discovered. The completed table from the student sheet appears below.

Number of blocks	Moves by A	Moves by B	Moves by C	Moves by D	Moves by E	Total # of moves (minimum)
1	1	0	0	0	0	1
2	2	1	0	0	0	3
3	5	3	1	0	0	9
4	14	9	3	1	0	27
5	41	27	9	3	1	81
n						3^{n-1}

After students have determined the number of moves by each block, they are challenged to try and find some other patterns in the puzzle. The following

are a few examples. When solving the puzzle, it is obvious that Block A moves every other time. Since it must always move first, this means it moves only on odd numbered moves. Therefore, the other blocks only move on even numbered moves. To illustrate, here is one way to record the pattern of moves for four blocks: A, B, A, B, A, C, A, B, A, B, A, C, A, B, A, B, A, D, A, B, A, B, A, C, A, B, A. It's easy to see that Block A moves on the each of the 14 odd numbered moves (1st - 27th) and that the other blocks only move on even numbered moves. Therefore, if the range of moves is known, then the number of moves Block A makes is the number of odd numbers within that range. This is shown in the column "Moves by A" in the previous table. For example, there are 5 odd numbers from 1-9 and 41 odd numbers from 1-81.

Other patterns emerge when you look at the numbered moves for the different blocks. The following list shows these patterns for the first four blocks.

One block:

A: 1

Two blocks:

A: 1, 3

B: 2

Three blocks:

A: 1, 3, 5, 7, 9

B: 2, 4, 8

C: 6

Four blocks:

A: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27

B: 2, 4, 8, 10, 14, 16, 20, 22, 26

C: 6, 12, 24

D: 18

With a bit of exploring, students may be able to discover patterns other than the ones listed here. It is important that you allow students ample time to work on this problem and that you provide some forum for students to share their discoveries with others in the class. One way to do this is to dedicate a bulletin board to the problem where students can put up their discoveries for others to see. Another is to have a class discussion and to act as the recorder for this session. Be sure not to shortchange this important time when students are able to share the things they learned in this lesson.

I hope that this *Maximizing Math* activity is worthwhile for you and your students. We will have another activity for you in the next issue of *AIMS*[®]. If you have any questions or comments, you can send me an email (dyoungs@fresno.edu) or write to me in care of AIMS.

Puzzle Solutions

Junk's Puzzle Solutions

One block (1 move)

1. A right

Two blocks (3 moves)

1. A right, down
2. B down, right, up
3. A up

Three blocks (9 moves)

1. A right, down
2. B down, right, up
3. A up, left
4. B down
5. A right, down
6. C down, right, up
7. A up, left
8. B up
9. A right

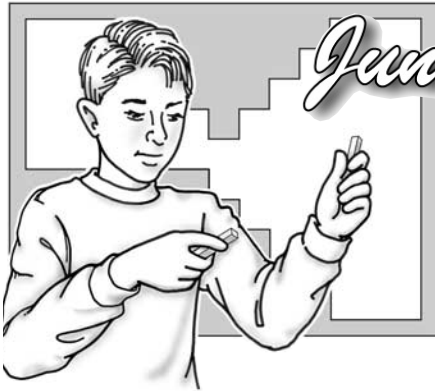
Four blocks (27 moves)

1. A right, down
2. B down, right, up
3. A up, left
4. B down
5. A right, down
6. C down, right, up
7. A up, left
8. B up
9. A right, down
10. B down, left, up
11. A up, left
12. C down
13. A right, down
14. B down, right, up
15. A up, left
16. B down
17. A right, down
18. D down, right, up
19. A up, left
20. B up
21. A right, down
22. B down, left, up
23. A up, left
24. C up
25. A right, down
26. B down, right, up
27. A up

Five blocks (81 moves)

1. A right, down
2. B down, right, up
3. A up, left
4. B down
5. A right, down
6. C down, right, up
7. A up, left
8. B up
9. A right, down
10. B down, left, up
11. A up, left
12. C down
13. A right, down
14. B down, right, up
15. A up, left
16. B down
17. A right, down
18. D down, right, up

19. A up, left
20. B up
21. A right, down
22. B down, left, up
23. A up, left
24. C up
25. A right, down
26. B down, right, up
27. A up, left
28. B down
29. A right, down
30. C down, left, up
31. A up, left
32. B up
33. A right, down
34. B down, left, up
35. A up, left
36. D down
37. A right, down
38. B down, right, up
39. A up, left
40. B down
41. A right, down
42. C down, right, up
43. A up, left
44. B up
45. A right, down
46. B down, left, up
47. A up, left
48. C down
49. A right, down
50. B down, right, up
51. A up, left
52. B down
53. A right, down
54. E down, right, up
55. A up, left
56. B up
57. A right, down
58. B down, left, up
59. A up, left
60. C up
61. A right, down
62. B down, right, up
63. A up, left
64. B down
65. A right, down
66. C down, left, up
67. A up, left
68. B up
69. A right, down
70. B down, left, up
71. A up, left
72. D up
73. A right, down
74. B down, right, up
75. A up, left
76. B down
77. A right, down
78. C down, right, up
79. A up, left
80. B up
81. A right



Junk's Puzzle: **THE MATHEMATICS**

To do this activity you will need the five blocks and frame from Puzzle 3 in the Puzzle Corner. What is the minimum number of moves it takes to solve this puzzle?

When mathematicians encounter difficult problems like this, they often try easier problems first to see if that helps. You will do that now.

Place Block A in the left side of the frame. What is the minimum number of moves it takes to get it to the right side of the frame? (See how easy this is!) Record your answer in the table below.

Place Blocks A and B in the left side of the frame. What is the minimum number of moves this time? Record your answer in the table.

Place Blocks A, B, and C in the left side of the frame and get them to switch. What is the minimum number of moves? Record your answer.

Can you see a pattern developing when you look at the number of moves it takes to switch the blocks? Look carefully at your answers for the first three levels and then try to predict the number of moves for four blocks.

Number of blocks	1	2	3	4	5
Minimum # of moves					

Try the puzzle with four blocks and see if your prediction was correct. Describe your findings below.

Junk's Puzzle: THE MATHEMATICS

Now that you have determined the minimum number of moves for 1 - 4 blocks, you should be able to determine the number of moves required to switch five blocks. Make your prediction and then do the puzzle to see if you were correct.

Describe the pattern you see in the number of moves it takes to switch different numbers of blocks.



Now that you have discovered the pattern in the minimum number of moves, try to find the patterns of moves for each block. Fill in the chart below with your information. The first two have been done for you.

Number of blocks	Moves by A	Moves by B	Moves by C	Moves by D	Moves by E	Total # of moves (minimum)
1	1	0	0	0	0	1
2	2	1	0	0	0	3
3						
4						
5						

What patterns do you notice in the above table?

Junk's Puzzle: THE MATHEMATICS

To see some other patterns, the following table lists the sequence of moves each block makes for one and two blocks. Complete the table for three blocks (nine moves).

Move	1	2	3	4	5	6	7	8	9
One block	A								
Two blocks	A	B	A						
Three blocks									

What patterns do you notice in this table?

What would the pattern be for four blocks?

What other patterns do you notice in the puzzle? List some of your discoveries here.

