ORIGAMI SQUARED

by Michelle Youngs

The Activity

This month the *Maximizing Math* and *Puzzle Corner* activities have been combined into one two-part activity. The challenge in the *Puzzle Corner* section is to use all five puzzle pieces to make a square and then determine if it is possible to make squares with fewer than five pieces. The *Maximizing Math* section has been divided into two parts. Depending on the age and abilities of your students, the second section may not be appropriate.

In the first part students are challenged to develop a plan that would allow them to determine the area of each puzzle piece if they were given the area value of only one piece of their choice. Once they have developed a plan, they should be given an area value by you (accurate or arbitrary) for the puzzle piece they have chosen and use that value to determine the areas of the other pieces. While this task sounds difficult, it is really a simple case of division and addition, once it is clear how the pieces are related to each other (see *Solutions* for more details).

In the second part, students are asked to determine and compare the perimeters and areas of the different squares which can be made using one or more puzzle pieces and justify their reasoning. This section requires more advanced reasoning and mathematics, including the understanding of square roots, and is not appropriate for younger students.

What makes this activity different from any which have been done in these columns before is that students will use origami to construct their own puzzle pieces rather than merely cut them out. This adds an additional challenge, both for the students and for you as the teacher.

Using origami in the classroom has many benefits that might not at first be apparent. Students must learn to follow written, pictorial, and verbal instructions to complete their folds and successfully make each piece. They will sharpen their fine motor skills as they concentrate on creating precise folds and accurate pieces. Often those students who are traditionally poor at math excel at origami because it uses a different part of the brain. Recently while doing some origami with a class of seventh graders I overheard the comment, "Hey, he's stupid at math, how come he's doing this better than me?" You may find that this activity will be an opportunity for your "low-achieving" students to excel and have success in a subject that they rarely do well in.

Essential tips for successful origami

1. Origami takes time. While you may find the folds easy, your students will not at first. Even with careful explanation, visual aids, and diagrams, students may struggle to get the folds right. This activity will probably have to be spread over several days or assigned to be taken home as homework. Students can fold their puzzle pieces on one day, work on the solution another day, and explore the math on yet another day.
2. Fold each puzzle piece yourself before doing the activity with your class. This allows you to understand the directions and enables you to give students help when they get stuck. It also gives students a final product to observe and compare to their own models for accuracy.
3. Have students work in pairs. If either student gets stuck, he/she can ask his/her partner for help before coming to you for clarification.
4. Choose your paper carefully. If your students have never done origami before, larger squares will be easier for them. Six-inch squares of origami paper can be purchased at most craft and stationery stores and from some educational suppliers such as Key Curriculum Press. There are also several internet sites that sell origami paper, such as www.shizu.com and www.origami-usa.org. For this particular activity, lightweight paper is the best. It is also good if the paper is a different color on each of the two sides so that students can follow the diagrams more easily. You will need five squares of paper (all the same size) for each puzzle your students will be making.
5. When you do the activity with your students you will almost certainly need to go through the

folding instructions step-by-step as a class. One effective way to do this is to fold a square of waxed paper on the overhead projector. Because waxed paper is translucent, students can see the fold lines (which show up black) as well as the shape of the folded paper and check these against their own folding at each step. Another method is to use a very large square of paper to demonstrate the folds.

6. Before you begin folding as a class, go over the sheets which explain the symbols used in the diagrams so that students understand what each one means.

Conclusion

This activity offers something for students at many different levels. They will be challenged to carefully follow directions and sharpen their fine motor skills as they fold their pieces; they will be challenged spatially as they try to arrange their pieces to form a square; and they will be challenged deductively as they determine the areas and perimeters of the different puzzle pieces and possible squares.

I hope you enjoy this slightly different problem and are willing to spend the time to make it work in your classroom. If you have any questions or comments, don't hesitate to write to me here at AIMS: P.O. Box 8120 Fresno, CA 93747, or by email: meyoungs@fresno.edu. We'll be back next month with two more activities for you and your students.

Credits

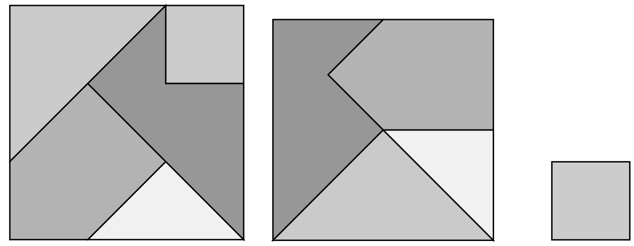
This model was designed by Marc Kirschenbaum under the name *Square Puzzle* (©1999) and is used with his permission.

Solutions

Because the *Puzzle Corner* and *Maximizing Math* have been combined into one activity, we will be forced to break the tradition and give the solutions to the puzzle in this issue so that the solutions to the math portion can be discussed. Please resist looking at these solutions until you have solved the puzzle for yourself.

Puzzle Corner

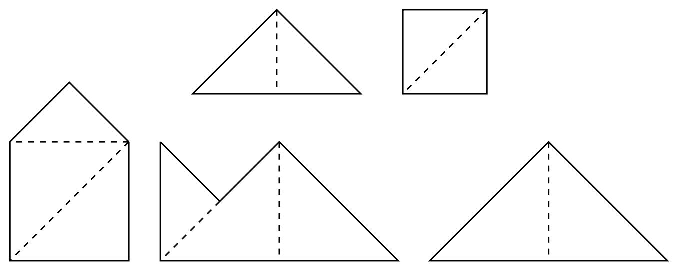
There is only one possible way to make a square using all five puzzle pieces. By using only four of the five puzzle pieces, a second, slightly smaller square is possible. If you also count the square puzzle piece itself, there are three possible squares that can be made by using different numbers of puzzle pieces. All three possibilities are shown below.



Square using all five pieces | Square using only four pieces | Square puzzle piece

Maximizing Math: Part One

In the first part of the *Maximizing Math* portion of this activity, students are asked to choose one puzzle piece and develop a plan that would let them determine the area of each piece, given the area of the one piece they chose.



As you can see, each puzzle piece can be divided into either two or three triangles. These triangles are two different sizes: congruent with the small triangular puzzle piece, or congruent with half of the small triangular puzzle piece. Notice also that the square puzzle piece and the small triangular puzzle piece can both be divided into two congruent triangles, indicating that they have the same area.

When the pieces are divided in this way it is easy to see that knowing the area of either the small triangular puzzle piece or the square puzzle piece would quickly allow you to determine the area of every other puzzle piece.

Let us assume that we know the area of the square puzzle piece to be six units^2. This means that the area of the small triangle is also six units^2, and one half of the small triangle (or the square) is three units^2. The large triangle would therefore have an area of 12 units^2 because it is made up of two small triangles, and the remaining two pieces would both have areas of 15 units^2 because they are each made up of two small triangles plus one half of the small triangle.

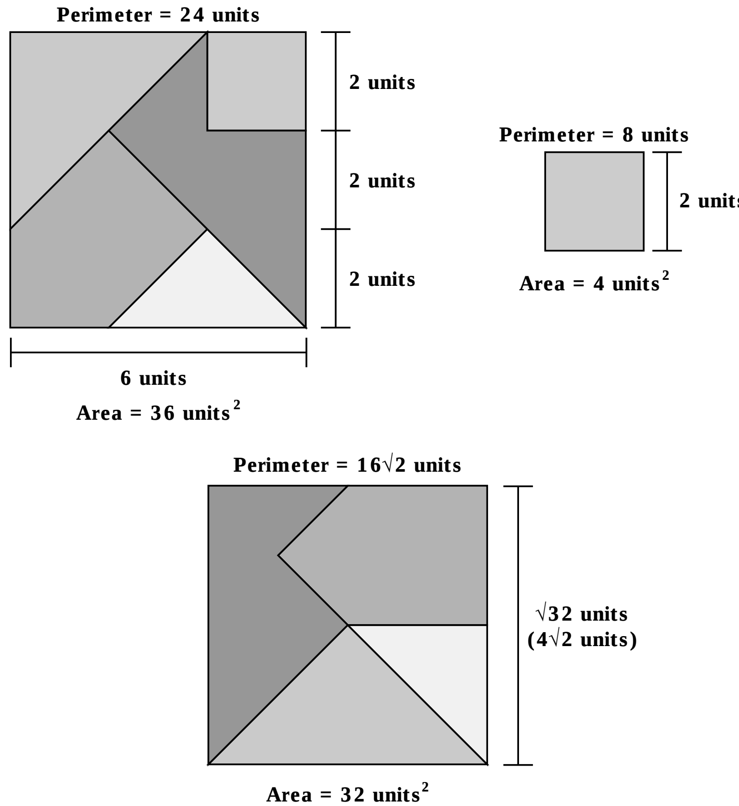
Maximizing Math: Part Two

In the second part of the *Maximizing Math* portion of the activity, students are asked to determine the area and perimeter of the three possible squares based on the knowledge that the perimeter of the largest square is 24 units.* If the perimeter is 24 units, this means that each side of the square is six units ($24 \div 4 = 6$), making the area 36 units^2.

Knowing this information allows us to determine the dimensions of the small square puzzle piece because, as you can see in the diagram, the small square piece has sides that are one third the length of the sides in the large square. Once it is determined that the square puzzle piece has sides that are two units each ($6 \div 3 = 2$), it quickly follows that it has a perimeter of 8 units and an area of 4 units^2.

With this knowledge, it can be determined that the square which uses only four pieces has an area of 32 units^2 because it does not use the small square puzzle piece (36 units^2 - 4 units^2 = 32 units^2). If the area of the four-piece square is 32 units^2, then the length of each side is $\sqrt{32}$ units, which can be reduced to $4\sqrt{2}$ units. This makes the perimeter of the smaller square $16\sqrt{2}$ units.

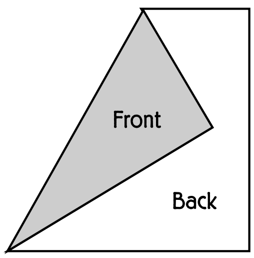
* Note: If you are using six-inch squares of paper, the units are the actual dimensions of the pieces in inches.



Key to Folding Symbols

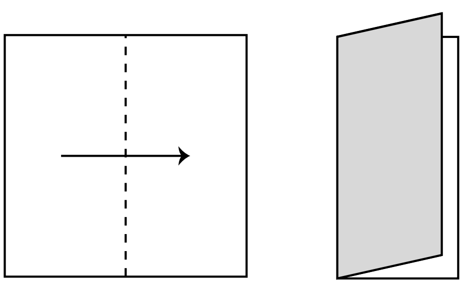
Paper

1. Traditional origami paper is square and often has a white side and a colored side. All of the puzzle pieces in this activity begin with the colored side of the paper facing up. Each diagram has been shaded to indicate front (colored) and back (white). Be sure to pay close attention to which side of the paper is being used or folded at any given time.

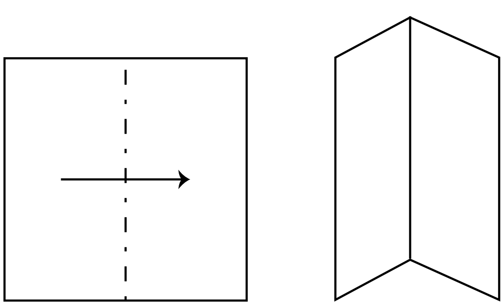


Folds

1. Valley folds are the most common kind of fold in origami. They are *concave* creases indicated by a dashed line.

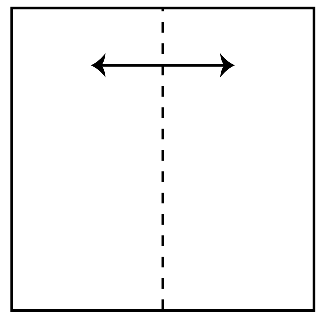


2. Mountain folds are not as common as valley folds in origami. They can occur both by themselves and in combination with a valley fold (see *Arrows #2*). A mountain fold is *convex* and is indicated by a dash-dot-dash line.

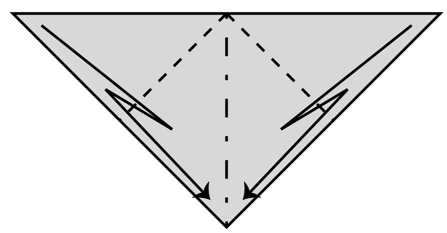


Arrows

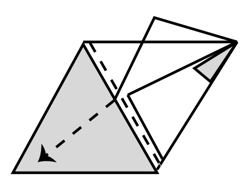
1. A double ended arrow indicates a *crease* that is to be folded and then opened up. A crease differs from a fold because the paper returns to its previous shape after being folded.



2. A jagged arrow indicates a combination fold where a mountain and a valley fold are both used. In these situations the paper is folded both in (valley) and out (mountain) in the same step.



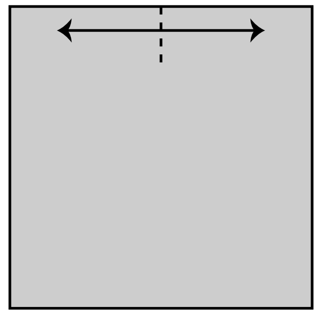
3. An arrow that is partially solid and partially dashed indicates that something is inserted at the point where the line becomes broken.



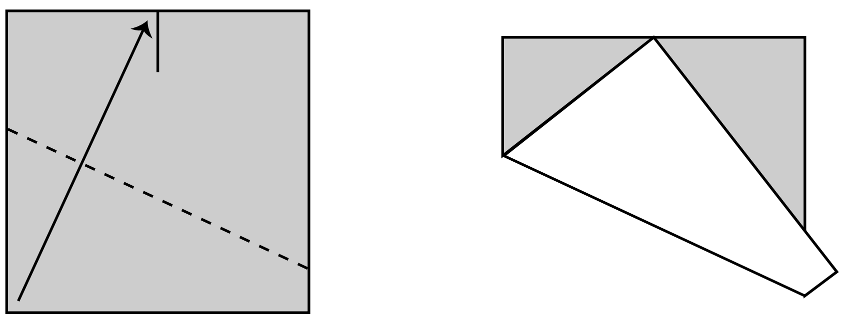
Origami Squared

Puzzle Piece One

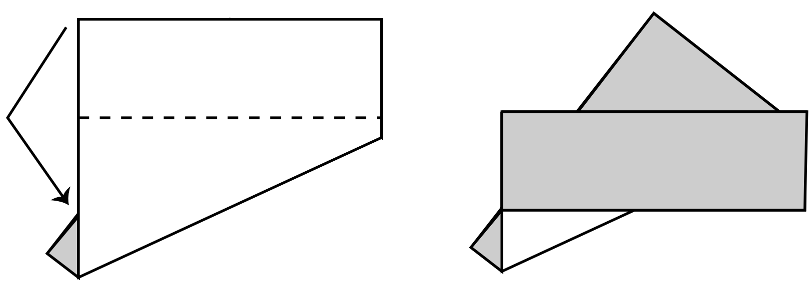
1. Start with your square of paper colored side up. Fold it in half vertically, making only a short crease at the top of the paper.



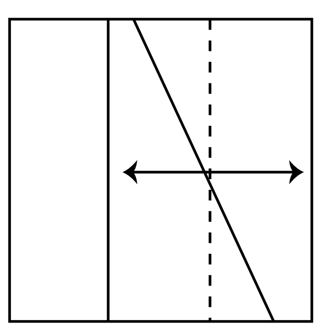
2. Fold the bottom left corner up so that it touches the top of the crease.



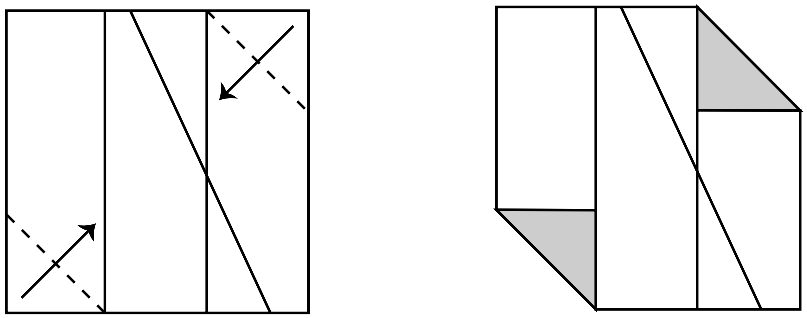
3. Flip the paper over and make a horizontal fold across the paper so that the top left corner meets the point where the front of the paper crosses the left edge.



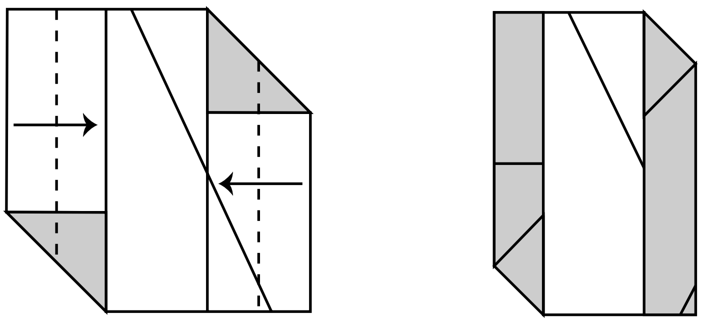
4. Unfold the paper so that the white side is face up, being sure to orient it as shown in the diagram. Fold the right edge in to meet the vertical fold line, crease, and unfold.



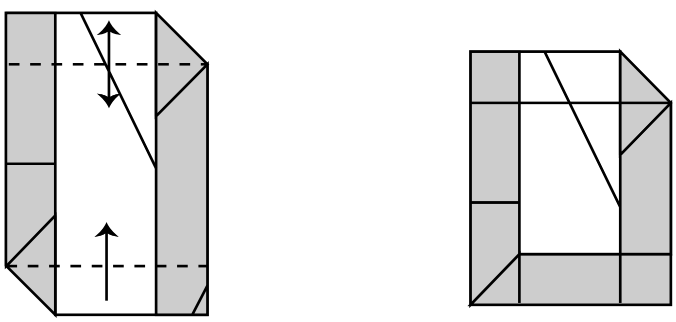
5. Fold the top right and bottom left corners in so that the top and bottom edges are flush with the nearest vertical fold lines.



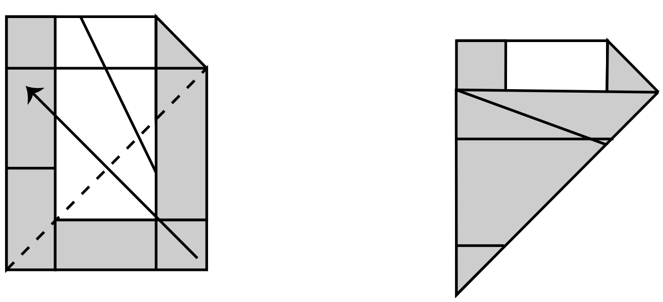
6. Fold both sides in so that the edges of the paper meet the nearest vertical fold lines.



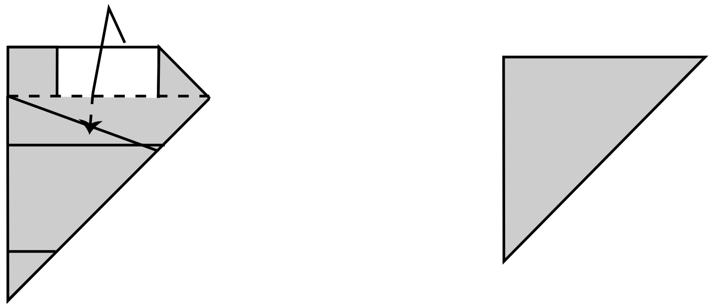
7. Crease the top of the paper horizontally as shown. Fold the bottom of the paper up in the same fashion.



8. Fold the bottom right corner up to the point where the top horizontal line meets the left side of the paper.

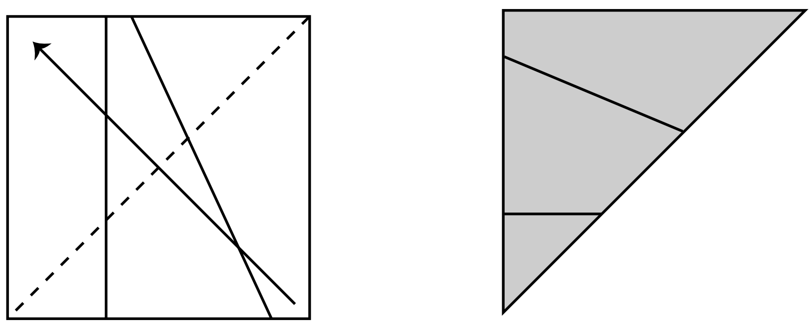


9. Tuck the top flap into the pocket created by your last fold. This is your first puzzle piece.

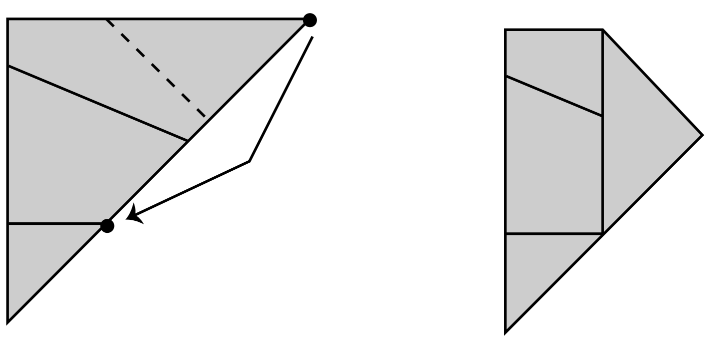


Puzzle Piece Two

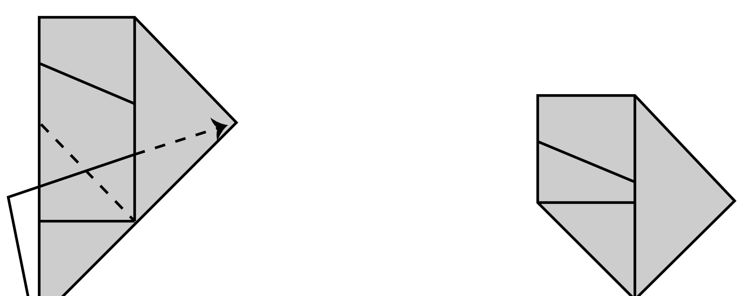
1. Repeat the first four steps from *Puzzle Piece One* with your second square of paper. Fold the paper in half along the diagonal, as shown.



2. Fold the top right corner down to meet the bottom fold line as indicated.

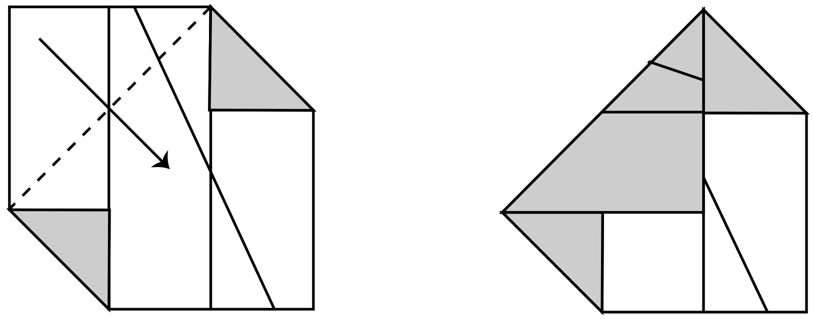


3. Fold the bottom left point up to meet the right point as shown. As you make the fold, tuck the point into the pocket. This is your second puzzle piece.

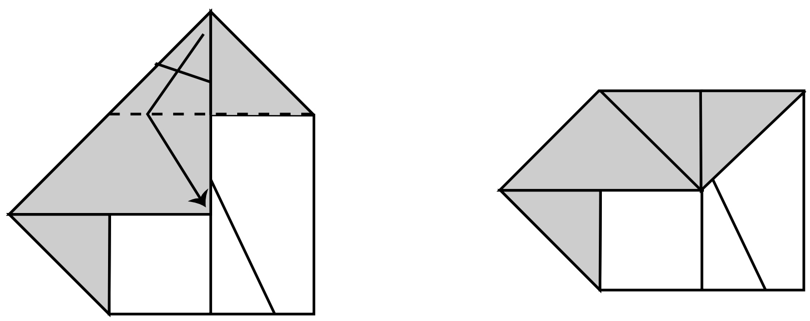


Puzzle Piece Three

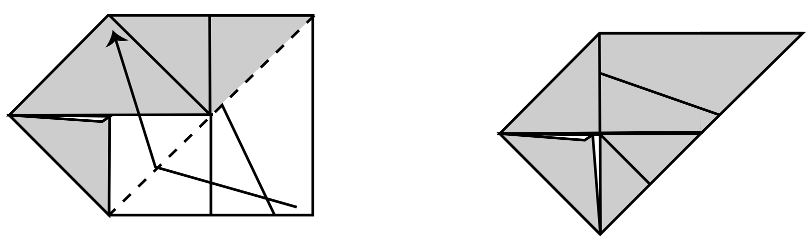
1. Repeat the first six steps from *Puzzle Piece One* with your third square of paper. Fold the top left corner in along the diagonal indicated.



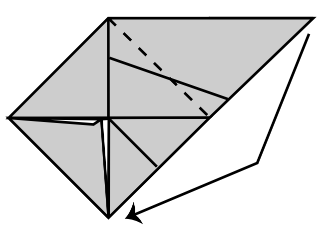
2. Fold the top of the paper down so that the point meets the point that you folded down in the previous step.



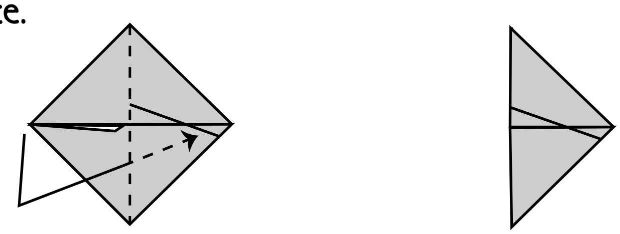
3. Fold the bottom right corner of the paper up as indicated.



4. Fold the top right corner down to meet the bottom point of the shape.

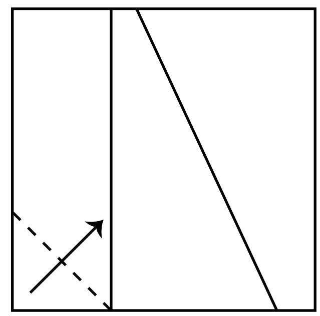


5. Fold the shape in half vertically, tucking the left side into the right pocket as you do. This is your third puzzle piece.

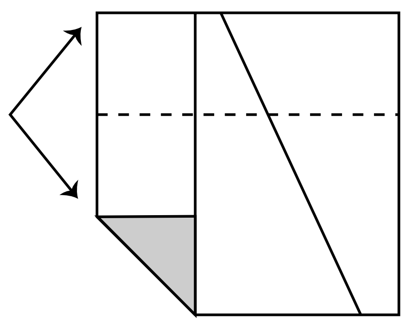


Puzzle Piece Four

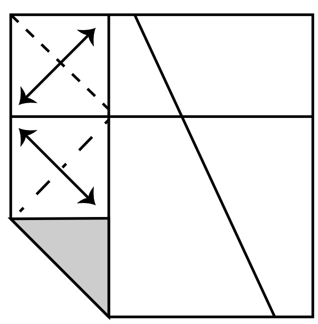
1. Repeat the first four steps from *Puzzle Piece One* with your fourth piece of paper. Fold the bottom left corner up so that the bottom edge is flush with the vertical fold line.



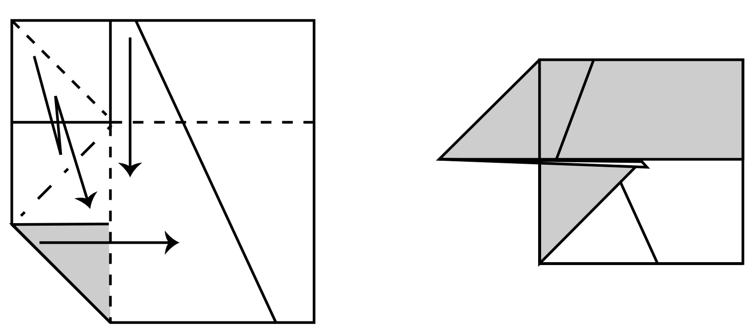
2. Fold the top of the paper down to meet the edge of the fold you just created, crease, and unfold.



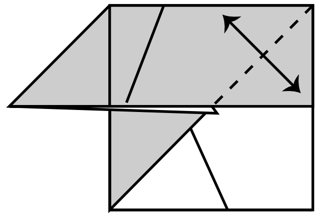
3. Make two creases on the left side of your paper as indicated. The upper of the two creases should be a valley crease, and the lower a mountain crease.



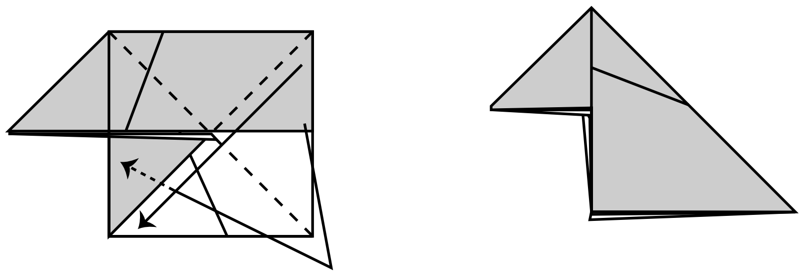
4. Using a series of simultaneous folds, bring the top of the paper down, while bringing the right side in. This should result in the shape shown at the right.



5. Crease the top layer of the paper as indicated.

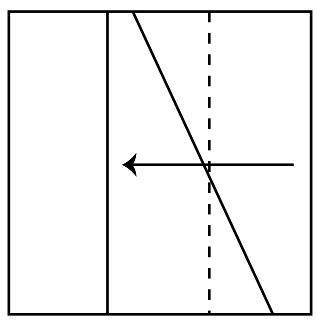


6. Fold the paper in half diagonally as indicated by the dashed line. As you fold, tuck the flap that you creased in the previous step into the triangular pocket as shown. This is your fourth puzzle piece.

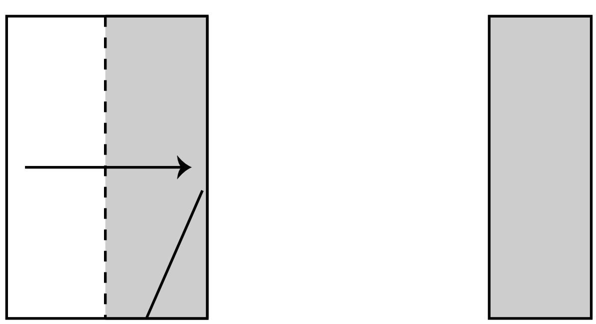


Puzzle Piece Five

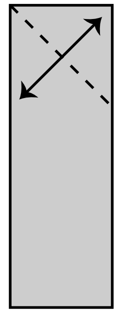
1. Repeat the first four steps from *Puzzle Piece One* with your fifth square of paper. Fold the right side of the paper in to meet the vertical fold line.



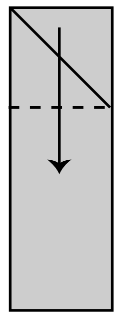
2. Fold the left edge of the paper over to meet the right edge.



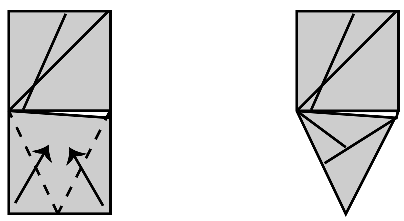
3. Crease the top of your paper as indicated so that the top of the paper is flush with the left side.



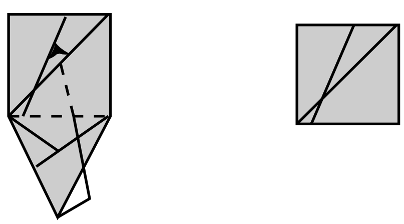
4. Fold the top of your paper down at the point where the diagonal fold line meets the right side.



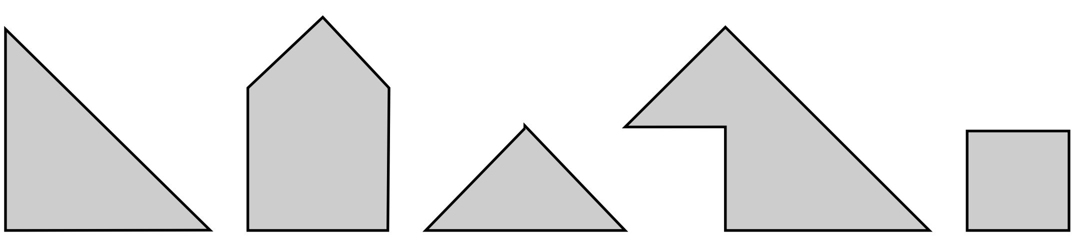
5. Fold the bottom portion of your paper in to make a triangular point as shown.



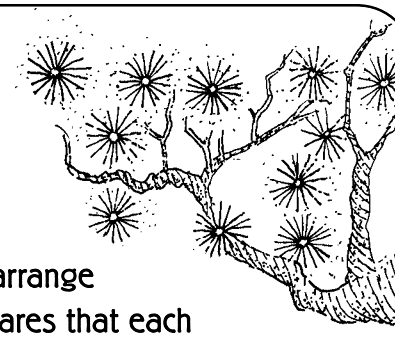
6. Tuck this triangular tab into the pocket formed by the previous folds. This is your completed puzzle piece.



These are the five puzzle pieces.

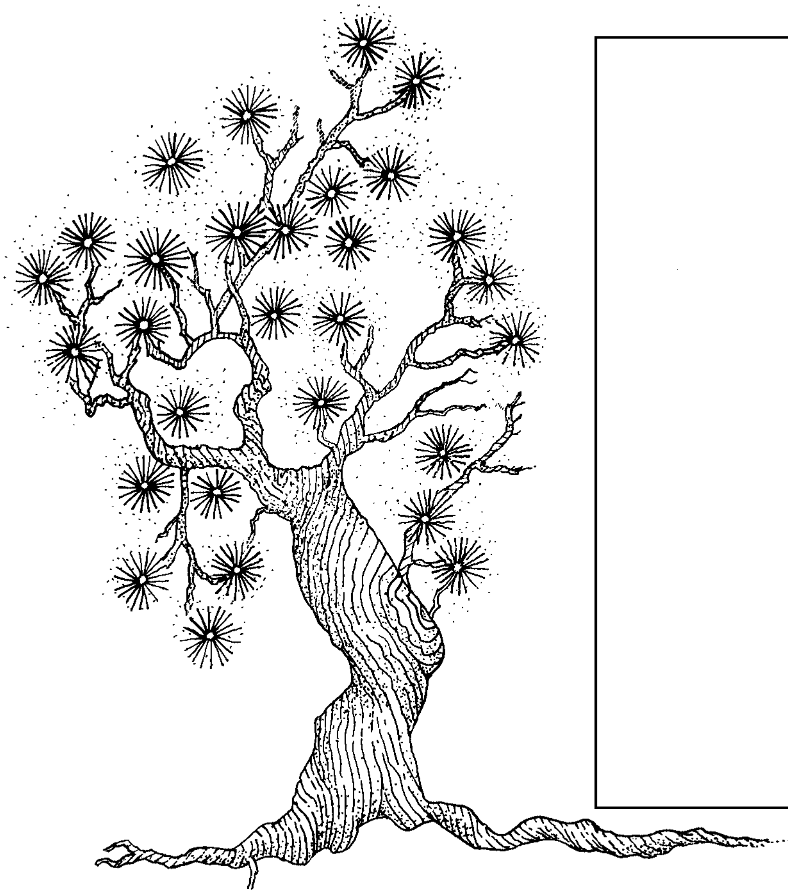


ORIGAMI SQUARED



Puzzle Corner

The object of this puzzle is to take the five pieces that you folded and arrange them to form a square. This square should be the same size as the squares that each piece was folded from. Once you discover a solution, draw it in the square below.



Is it possible to make a square using less than five pieces? How? How many different squares are possible using one or more of the puzzle pieces? Draw a picture of each additional solution you discover in the spaces provided.

ORIGAMI SQUARED

Maximizing Math: Part One

If you could be told the area of any one puzzle piece and had to use that information to determine the areas of the other four puzzle pieces, which piece would you choose? Explain your reasoning using words and/or pictures. Show how you would use this piece to determine the area of each other piece.

Use the area value that your teacher gives you for the puzzle piece you have chosen to determine the area of each other puzzle piece. Show your work below.

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Maximizing Math: Part Two

Once you have solved the puzzle, fill in the table below with the required information for each possible square that can be made using one or more of the puzzle pieces. The perimeter for the five-piece solution has been given to get you started. Show your work in the space below the table using words and/or diagrams.

# of pieces in square	Perimeter in units	Length of each side	Area in units^2
5	24		

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Maximizing Math: Part Two

Once you have completed the table on the previous page, answer the questions below.

1. If the perimeter of the square that is made with all five pieces is 24 units, what is the length of each side? What is the area of that square? How were you able to determine this?

2. How does this information let you determine the area and perimeter of the small square puzzle piece?

3. What is the area of the square that can be made with four of the five puzzle pieces? How do you know?

4. What is the length of one side of this square? ... the total perimeter? Why?

5. What is the difference in area between the square that uses five puzzle pieces and the square that uses four puzzle pieces? Explain your reasoning.