Puzzle Corner/Maximizing Math Switching Sequences

by Michelle Pauls

The *Puzzle Corner* and *Maximizing Math* activities this month are two parts of one problem—a variation of a teaser that appeared in the March, 2003 *Games* magazine. The original version describes a five-volume set of books that have been reshelved in descending order, from five to one. The challenge is to put the books in order from one to five using the fewest possible number of moves. Each move consists of changing the location of two adjacent books. Any gap created by moved books is closed immediately.

Our version uses number cards for the numbers one through four, and offers the five-number version as an extension. While a single solution was given in *Games* for the original version, it turns out that there are multiple solutions, both to the five-number version and to our four-number variation. Additionally, there are many patterns in the solutions to be explored, making this a rich and valuable problem.

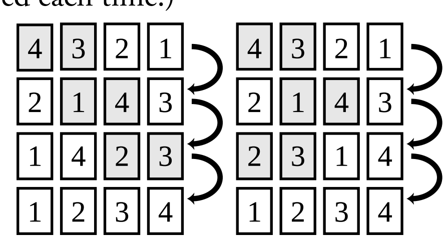
For the *Puzzle Corner* portion of the activity, students are challeged to take number cards arranged to read 4 3 2 1 and rearrange them to read 1 2 3 4 in the fewest number of moves. Each move must take two adjacent cards and place them in a different location. Whenever cards are moved, any gap they leave behind is closed. Once the minimum number of moves is determined, the challenge becomes to find every possible solution using that number of moves. I would encourage you to try this problem for yourself right now. Make four cards numbered 1, 2, 3, and 4, and arange them in descending order. See how many moves it takes to you put them in ascending order, each time moving two adjacent cards together. This will allow you to experince the joy of problem solving as well as give you a better understanding of what your students will go through when completing this problem.

Hopefully you were able to reorder your number cards in three moves—the minimum possible. Once this discovery of the minimum number of moves has taken place, the next challenge is to discover every possible way to re-order the cards in this number of moves. A page for recording solutions is provided, but there are fewer spaces than the total number of solutions. Students can either use the back of the paper for further recording, or you can make additional copies of the recording page available. It may be necessary to have students work on this problem over the course of several days in order for them to come up with all of the solutions.

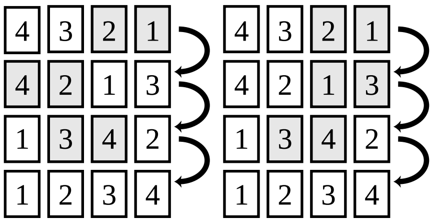
Once students believe they have found all of the possible solutions, they will be ready to move on to the *Maximizing Math* portion of the activity. Begin by having students get into small groups and compare their solutions. As a group, they should develop a list that includes all solutions found among them. Distribute the student pages and give students sufficient time to respond to the questions and complete the table of solutions. Close with a time of whole-class discussion where groups share their solutions and discuss the patterns they see.

Discussion of Patterns

There are many fascinating patterns that can be explored in the solutions. Of the 16 possible solutions, only four are truly unique in the sense that they move different number cards to produce different orders of numbers each move. Of the remaining solutions, four pairs move the same two cards for each move, but with different number orders resulting. The other four pairs move different number cards, but result in the same number orders. For example, in the solutions below, the same combinations of numbers are moved—four and three, then one and four, and finally two and three. However, the third number order produced by these moves is different—2 1 4 3 to 1 4 2 3 and 2 1 4 3 to 2 3 1 4. (The shaded spaces indicate the number cards that were moved each time.)



In the following solutions, the reverse is true. Different number cards are moved during one of the turns, but the same number sentences are produced each time.



It is fairly simple to prove that you have indeed found all possible solutions. If you begin with the starting number order of 4 3 2 1, you can determine all of the possible combinations that can result by moving two cards at a time. These possibilities are illustrated in the table below.

Beginning with 4 3 2 1

Move #s	Possible Results
2 1	2 1 4 3 4 2 1 3
3 2	3 2 4 1 4 1 3 2
4 3	2 1 4 3 2 4 3 1

Notice that two of the possible results are duplicates of each other—the order 2 1 4 3 can be reached by moving either the two and the one or the four and the three. This means that there are only five possible number orders which can result from a single move of the beginning sequence.

If you now repeat this process beginning with the ending order of 1 2 3 4, you will generate another five possibilities, outlined in the table below. (Again, two of the orders are the same.)

Beginning with 1 2 3 4

Move #s	Possible Results
1 2	3 1 2 4 3 4 1 2
2 3	2 3 1 4 1 4 2 3
3 4	3 4 1 2 1 3 4 2

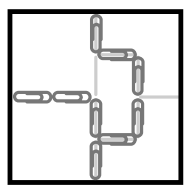
To obtain every possible solution, all that is necessary is to take one of the two lists generated, and repeat the process used to generate the list. That is, take each new sequence, and determine all of the possible sequences that could result from it by moving two numbers.

For example, the sequence 3 1 2 4 could become 2 3 1 4 or 2 4 3 1 by moving the three and the one, or 3 2 4 1 or 2 4 3 1 by moving the two and the four. (You would not move the one and the two because those were the numbers moved to generate the sequence.) Of those three unique possibilities, two—2 4 3 1, and 3 2 4 1—are in the list of results found in the first table. This means that it is possible to go from the beginning sequence to the solution using each of these two sequences.

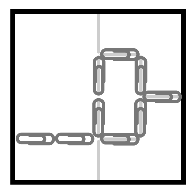
These are just some of the intriguing patterns that your students can discover while working on this problem. As an extension, repeat the problem using the numbers one through five and compare the results.

Last Month's Puzzle

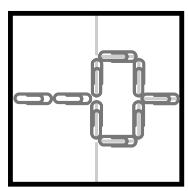
Penning Ponies started with 16 ponies divided into four pens containing eight, three, three, and two ponies respectively. Students were challenged to move only two paper clips (fence sections) to get different numbers of ponies contained in three-pen arrangements. One possible solution for each arrangement is shown below.



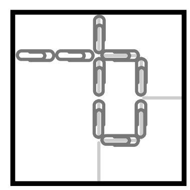
Four, six, and six ponies



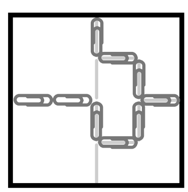
Two, five, and nine ponies



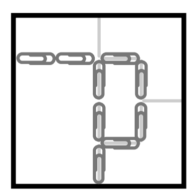
Two, seven, and seven ponies



Two, two, and 12 ponies



Three, six, and seven ponies



Two, six, and eight ponies

Switching Sequences

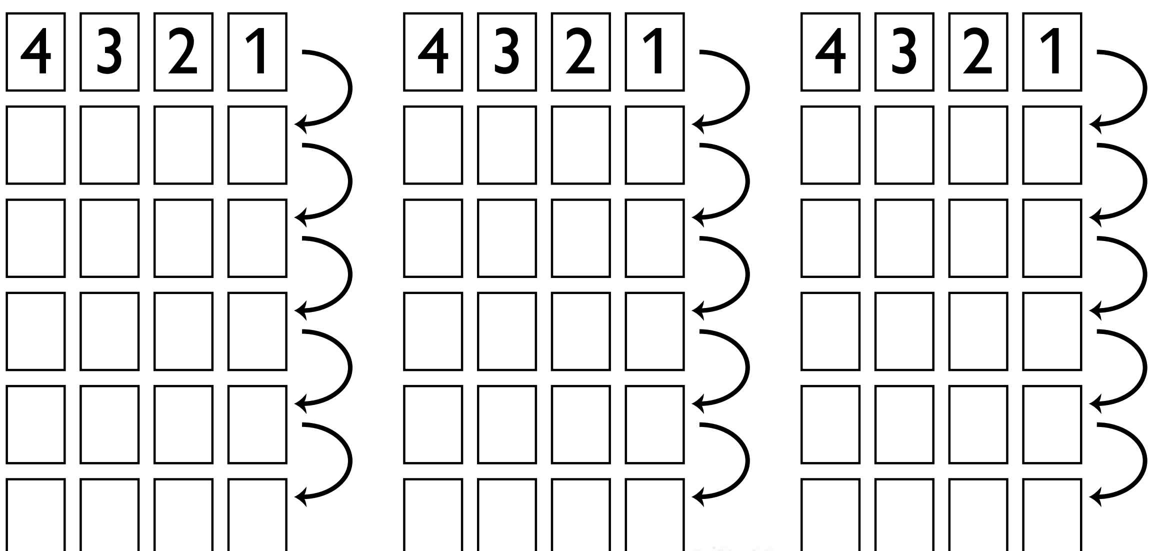
Puzzle Corner

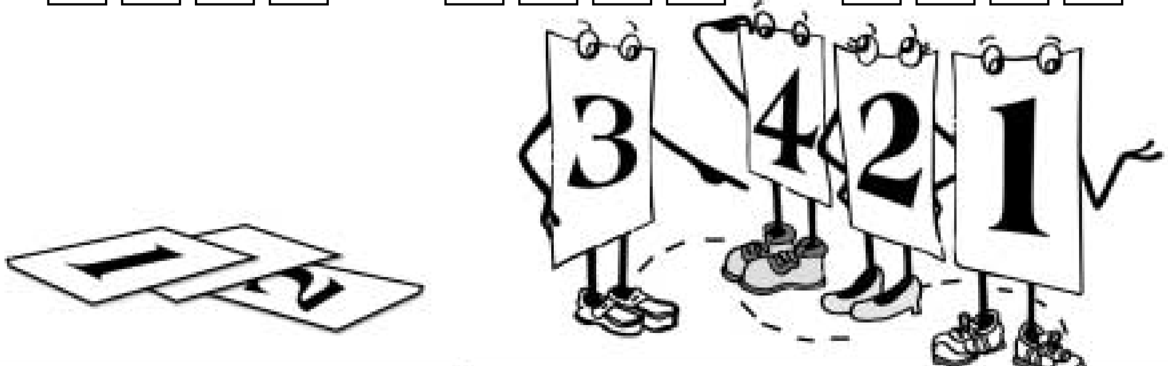
Lay your number cards in a row so that they read 4 3 2 1. Your challenge is to rearrange the cards so that they read 1 2 3 4 using the fewest number of moves. You must always move two cards at a time. The two cards moved must be next to each other. Whenever cards are moved, any gap they leave behind is closed.

Try it now and count how many moves you make. Record this number below.

Try it again, and see if you can do it in fewer moves. Keep trying until you can't do it in any fewer moves. What is the fewest number of moves it takes to re-order the numbers?

Once you find the fewest number of moves possible, see how many ways you can rearrange the cards in this many moves. Record each solution in the spaces provided here and on the next page. Shade the number cards you moved each time and write the new arrangement that resulted in the spaces below.





Switching Sequences

Puzzle Corner

To record your solutions, shade the number cards you moved each time and write the new arrangement that resulted in the spaces below. If you need more room, use the back of this paper.

4	3	2	1

4	3	2	1

4	3	2	1

4	3	2	1

4	3	2	1

4	3	2	1

Switching Sequences

Maximizing Math

1. What is the fewest number of moves in which you can rearrange four numbers? How do you know?

2. How many solutions did you discover? Do you think you have found them all? Why or why not?

3. Look at your solutions and describe any patterns or interesting things you notice.

4. Arrange the cards 4 3 2 1. Record every possible order that could result from moving two adjacent cards.

5. Arrange the cards 1 2 3 4. Record every possible order that could result from moving two adjacent cards.

6. Using these two lists, describe how it is possible to determine every solution for this problem.

Switching Sequences

Maximizing Math

Fill in the table below with each possible solution. Organize the solutions in a logical way and explain your organization below.

Starting Order	Move Cards	Second Order	Move Cards	Third Order	Move Cards	Final Order
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4
4 3 2 1						1 2 3 4

How I organized my solutions: