

**Friedrich Froebel**  
April 21, 1782 - June 21, 1852

# Quotes

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## *Froebel Style*

*A correct comprehension of external, material things is a preliminary to a just comprehension of intellectual relations.*

*The A, B, C of things must precede the A, B, C, or words, and give to the words (abstractions) their true foundations.*

*Perception is the beginning and the preliminary condition for thinking. One's own perceptions awaken one's own thinking in later stages of development.*

*Every child brings with him into the world the natural disposition to see correctly what is before him, or, in other words, the truth. If things are shown to him in their connection, his soul perceives them thus as a conception. But if, as often happens, things are brought before his mind singly, or piecemeal, and in fragments, then the natural disposition to see correctly is perverted to the opposite, and the healthy mind is perplexed.*

*The correct perception is a preparation for correct knowing and thinking.*

*No new subject of instruction should come to the scholar, of which he does not at least conjecture that it is grounded in the former subject, and how it is so grounded as its application shows, and concerning which he does not, however dimly, feel it to be a need of the human spirit.*

— Friedrich Froebel  
(1782-1852)

*Without an accurate acquaintance with the visible and tangible properties of things, our conceptions must be erroneous, our inferences fallacious, and our operations unsuccessful.*

*The truths of number, of form, of relationship in position, were all originally drawn from objects; and to present these truths to the child in the concrete is to let him learn them as the race learned them.*

*If we consider it, we shall find that exhaustive observation is an element of all great success.*

— Herbert Spencer  
(1820-1903)

*Instruction must begin with actual inspection, not with verbal descriptions of things. From such inspection it is that certain knowledge comes. What is actually seen remains faster in the memory than description or enumeration a hundred times as often repeated.*

— John Amos Comenius  
(1592-1670)

*Observation is the absolute basis of all knowledge. The first object, then, in education, must be to lead the child to observe with accuracy; the second, to express with correctness the results of his observation.*

— Johann Heinrich Pestalozzi  
(1746-1827)

# Geometry

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## Kindergarten Geometry— Froebel Style by Richard Thiessen

**G**eometry is a language” suggests a metaphor with implications for when and how we teach many of the concepts and relationships of geometry. Do we wait until high school geometry or do we create an environment in which children, beginning in kindergarten and continuing through the grades, acquire a large number of the concepts, relationships, and accompanying language of geometry? **If we wait until high school, students are forced to learn geometry as essentially a foreign language; however, if we begin early and if we are consistent in engaging students in thinking about and using the language of geometry, they can acquire geometry naturally as a subset of their first language.** In a three-part series of articles concluding in the May/June issue of AIMS, we argued for the importance of an early introduction and continued growth of geometry as a language throughout a child’s school experience.

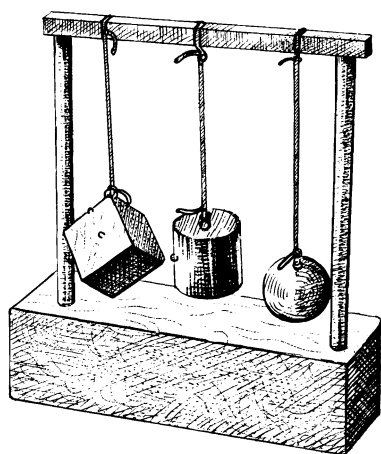
The purpose of this article is to introduce you to some of the work and beliefs of Friedrich Froebel, the inventor/founder of the kindergarten. This will further serve as an introduction to a series of articles that will focus on the philosophy, materials, and activities that Froebel used to introduce young students to the ideas and language of geometry. It was Thomas Banchoff, a mathematician who presently serves as the president of the Mathematical Association of America, who, in writing about the importance of an early introduction to teaching geometric ideas to young children, provided me with an introduction to Froebel. Banchoff described Froebel’s philosophy as follows: *If children could be stimulated to observe geometric objects from the earliest stage of their education, these ideas would come back to them again and again during the course of their schooling, deepening with each new level of sophistication. The rudimentary appreciation of shapes and forms at*

*the nursery school level would become more refined as students developed new skills in arithmetic and measurement and later in more formal algebra and geometry.* (Banchoff, p. 11)

Along with the establishment of the kindergarten, Froebel developed curricular materials and accompanying manipulatives that were based on his own philosophy and understanding of child development and learning. He referred to the activities as “occupations” and the manipulatives were called “gifts.” Geometric concepts were the focus of nearly 20 of these occupations and gifts which he saw as appropriate not only for children of what we would consider kindergarten age, but for children ranging in age from two or three to seven or eight. The geometric gifts, six or eight of which we will describe in some detail in this series of articles, were constructed primarily of wood. Gift number two included a cube, a cylinder, and a sphere.

The kindergartens that he established and those established later by his followers were provided with sets of the geometric gifts which were crafted by local woodworkers.

In the middle of the turmoil in mathematics education today, I find it refreshing and insightful to explore the life, beliefs, work, and passions of Friedrich Froebel. More than 150 years ago he was advocating the early introduction of the ideas and language of geometry to young children. Not only did he develop a curriculum for young children that had such a strong geometry component, he also developed, wrote about, and taught a philosophy and understanding of how young children come to know and understand geometric ideas and how these ideas relate to a deeper understanding of the world around them. As I read some of the many things written about what the mathematics education of children should or should not look like today, I find myself asking, what background does this writer bring to the discussion? Does this person really know something about mathematics? How much does this person know about children and about how children come to know mathematical ideas? What does this person believe about how teachers should be prepared if they are to effectively teach and help children acquire the ideas and language of mathematics? As we explore in this series of articles some of Froebel's ideas about teaching geometry to young children, I think it is important and interesting to know something of his story, to know how some of the questions I have just raised would be answered if they were asked about him. While there is much more to his story than can be told in one short article, I have chosen pieces of his story that I believe are especially relevant.



Froebel's life spanned the years 1782 to 1852. His mother died when he was nine months old and as a result his early childhood was a most unhappy time. He was raised by his father, a busy country clergyman with little time for his youngest son. One biographer says of these years of Froebel's childhood that he was *shut up in the gloomy parsonage most of the time and left to the care of the single housemaid and his own devices, he seems to have lacked not only playfellows, but also playthings.* (Blake, p. 218) In reflecting back on this period of his life, Froebel himself describes the solitude, the need he felt for companionship, and for things with which to play. *He describes an incident that occurred during these years. He became greatly interested in watching some workmen who were repairing the neighboring church, and that a strong desire took hold of him to undertake the building of a church, and that he began to collect sticks and stones as heavy as he could carry for such a structure. His impulse was to use such pieces of furniture or other objects as he could secure with which to imitate the real builders. But his efforts ended in utter failure, and in giving an account of his experiment he says he remembers very well that even at that early age he thought that children ought to have suitable materials and somebody to show them how to go to work with it, so that they might attain better results.* (Blake, p. 218) In commenting on this incident, one of Froebel's biographers says, *Who can fail to see that in this incident, which made such a deep impression on the boy's mind, lay the germ of his endeavor, later in life, to devise the gifts and occupations of the kindergarten?* (Blake, p. 218)

When he was ten years old, Froebel went to live with his uncle. Life in this new setting was dramatically different. He says of these days, *As austerity reigned in my father's house, so here kindness and benevolence. I saw there, in respect to myself, distrust; here, confidence; there I felt constraint, here freedom. While there I had been hardly at all among boys of my own age; here I found certainly as many as forty fellow-pupils, for I*

*entered the higher class in the town school.* (Blake, p. 222) He goes on to say that in the new school he was well taught and he discovered that, *Mathematics lay near my nature. When I received private instruction in this branch my advance steps were so marked that they bordered on the height of knowledge and ability possessed by my teacher, which was by no means slight.* (Blake, p. 222)

At the age of 17, Froebel entered the university thinking that he wanted to study architecture. However, as he took courses in mathematics and the natural sciences, he found himself drawn to them rather than to those more directly related to architecture. He says of his teachers in botany, zoology, and mineralogy, that they were *sensible, loving and benevolent and that through them his insight into nature was essentially quickened and his love for observing it made more active.*

While still pursuing the course of study in architecture, Froebel encountered a Dr. Gruner who was the principal of a new teacher training school. Gruner challenged him to consider becoming a teacher. He said, *My friend you should not be an architect, you should be a school master. There is a place open in our school; if you agree to it the place is yours.* (Balke, p. 228)

Froebel accepted the challenge, but before moving to the new school, he went to Switzerland, where for two weeks he sat under the teaching of Johann Heinrich Pestalozzi *who was the great educational light of the day, the fountain-head of all new educational ideas.* (Blake, p. 228) Thus began his pilgrimage as a teacher, which he would leave and come back to several times over the next ten or 12 years. Of his early experiences as a teacher, Froebel said, *Even in the first hour of teaching, it did not seem strange to me. It appeared to me as if I had already been a teacher and was born to it. It is plain to me now that I was really fitted for no other calling, and yet I must tell you that never in my life had I thought to become a teacher. In the hours of instruction I feel myself as truly in my element as the fish in the water or the bird in the air.* (Blake, p. 228)

Along with his teaching at the school, Froebel contracted with a wealthy family to spend two hours each day tutoring three of the children. This proved to be such an insightful experience that after only two years at the school he decided to devote full time to the tutoring of the three children. He recalls how he went about engaging the children in a variety of physical activities, always with the goal of stimulating questions that would lead them to learn from the activities.

While he was an astute observer of children and a gifted teacher, Froebel felt there was much more he wanted to know about children, about their nature, about their intellectual development, and about how they learn. In 1808, he was given the opportunity to move, with his three charges, to work with Pestalozzi at his school in Switzerland. Here Froebel became at once both student and teacher. While spending time listening to and interacting with Pestalozzi, he continued to tutor his three students. During the two years that they spent with Pestalozzi, Froebel began to formulate his own understanding and theory of child development and learning which in many respects differed from that of his mentor and teacher.

In the summer of 1811 at the age of 29, Froebel apparently sensed that he needed to go back to complete the university studies which he had left behind in order to embark on a teaching career. Rather than architecture, however, this time he concentrated on a course of study that included mineralogy, geology, and crystallography. It was this training that two or three years later found him in the position of curator of a mineralogical museum in Berlin. During the year or two that he spent at the museum, he continued his studies of mineralogy and crystallography as well as the history of ancient philosophy. During this time he also gave some thought to perhaps teaching at the University. He rejected this idea, however, and gave as his reason, *The opportunities I had of observing the natural history students of that time, their very slight*

*knowledge of their subject, their deficiency of perceptive power, their still greater want of the true scientific spirit, warned me back from such a plan.* (Blake, p. 232)

Having rejected the idea of a professorship, Froebel was again drawn to the teaching of children and was offered the opportunity to work at a small, struggling elementary school. He remained at the school for 14 years as teacher and principal. It was during these years that his study of children and thinking about what was wrong with the educational system of his day brought him to the conviction that what was needed was a reform of early childhood education that would provide a proper foundation for future development and learning. It was out of this concern that the idea of an institution for little children was born. In 1840, Froebel established the first kindergarten which would soon have associated with it a training school *where women teachers could be shown how to deal with little children up to the age of seven.* (Blake, p. 244)

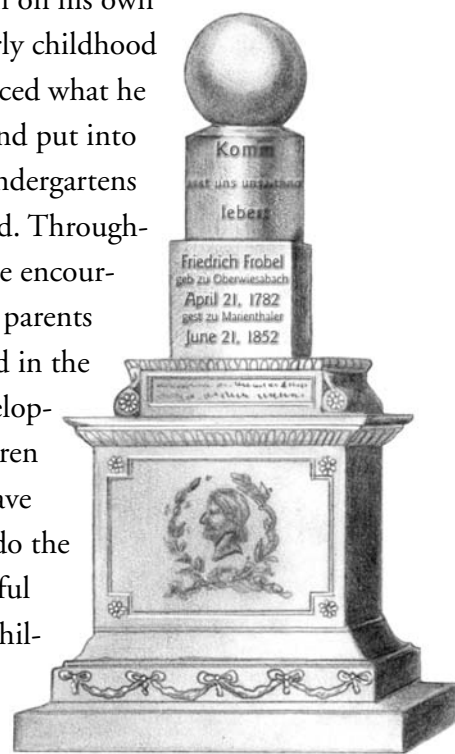
An interesting story associated with the establishment of this school is the naming of this institution for the teaching of young children. A colleague describes an afternoon walk with Froebel in which he kept repeating, *Oh, if I could only find a name for my youngest child.* The colleague goes on to say, *Suddenly he stood still as if riveted to the spot, and his eyes grew wonderfully bright. Then he shouted to the mountain so that it echoed to the four winds, "Eureka, Kindergarten shall the institution be called."* It was literally a mountain moment in his life, a brief period of inspiration which counted for more than months of everyday existence. (Blake, p. 244) Kindergarten can, of course, be translated "children's garden" or "garden of children."

Over the next 12 years until his death in 1852, Froebel founded a movement that swept over Europe and within only a few years after his death would find its way to the United States. During these last few years he continued to formulate, refine, teach, and write about his beliefs and philosophy with regard to

the early education of children. He also wrote two books that outline his philosophy and beliefs. Moreover, he left behind a devoted and loyal group of followers who continued to spread his vision for early childhood education and who were the ones responsible for that vision gaining acceptance in the United States.

The geometric gifts, even in his lifetime, became a symbol for his philosophy and beliefs about early childhood education. Froebel's tombstone is a base above which are stacked a cube, a cylinder, and a sphere. These are the geometric objects that make up gift two.

As I reflect over his life, it's as if all of the many varied experiences of the first 62 years were designed to prepare him for the vision and work in which he so passionately engaged during his final 12 years. While much of what Froebel came to believe about learning and intellectual development was influenced by what he learned as a student at the university, as a student under Pestalozzi, and as a teacher over a period of some 20 years, it is apparent from his story that it was his own intense observation of young children and his deep reflection on his own experiences in early childhood that most influenced what he came to believe and put into practice in the kindergartens that he established. Throughout his writing, he encourages teachers and parents who are interested in the learning and development of the children for whom they have responsibility to do the same—to be careful observers of the children in their care



and to reflect back on their own learning experiences as young children.

As we pursue this series of articles over the next several months, we will have the opportunity to be more specific about Froebel's philosophy and beliefs, and in particular will focus on his geometric gifts. The next article in the series will continue with the story of how the kindergarten became established in the United States and the stories of two men who assured that Froebel's curriculum and the geometric gifts that were central to that curriculum would be readily available to the kindergartens in this country.

### **References and Related Reading**

Banchoff, Thomas. "Dimension." In Steen, Lynne Arthur (Ed.): *On the Shoulders of Giants*. National Academy Press. Washington, DC. 1990.

Blake, Henry W. "Life of Friedrich Froebel." In Bradley, Milton (Ed.): *Paradise of Childhood*. Milton Bradley Co. Springfield, MA. 1923.

Brosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

# Geometry

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## *Froebel's Ideas Come to America* by Richard Thiessen

**T**he story of how Friedrich Froebel's kindergarten movement came to America and spread across the country during the last half of the 19th century is not complete without the stories of two men, Edward Wiebe and Milton Bradley. Wiebe is important to the story because he was one of the first to carry the idea of Froebel's kindergarten from Germany to America. Bradley's contribution was to spread the idea across the country through the publishing, manufacture, and marketing of Froebel's kindergarten materials. As it happened, the paths of these two men crossed in Springfield, Massachusetts in the mid-1860s.

Milton Bradley arrived in Springfield in 1856, a young man, 20 years of age, seeking an opportunity. Moving there from his childhood home in Maine, Bradley believed in himself and his skill. Trained in high school and trade school as a drafter, he obtained work with a company that manufactured locomotives and railroad cars. While working for this company he became aware of the need for the services of a person who could help individuals prepare and present inventions to the United States Patent Office. He decided to go into business for himself. The lettering on the door of his small place of business read:

Milton Bradley  
Mechanical Draftsman and Patent Solicitor.

While the business had a difficult time getting started, it was finally successful and led him to yet another venture that involved the lithographic process. This, in turn, led to the production of a game that he called *The Checkered Game of Life*. The sale of the game was surprisingly successful and set Bradley on a course that quickly changed the focus

of his business to that of creator, innovator, and manufacturer of games. By the time Edward Wiebe arrived in Springfield in the late 1860s, the Milton Bradley Company was successfully manufacturing and marketing an extensive line of games, toys, and, what Bradley liked to call, *home amusements*.

Edward Wiebe appeared in Springfield in about 1868. The sign on his house, which was near the Bradley home, read:

Professor Wiebe, Instructor in Music.

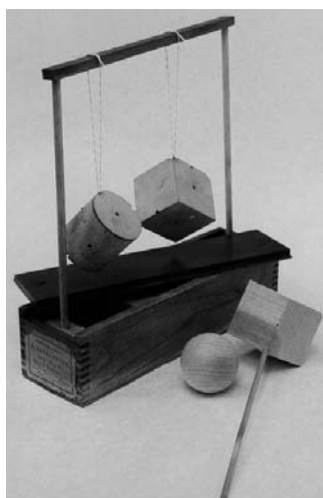
A stout, friendly man with a distinct German accent, he was known as quite a character about the city. With a love for children and music, Wiebe gave music lessons to many of the children of Springfield. Describing Wiebe's efforts to teach music to the children of Springfield, Shea, the author of a biography of Bradley, states, *Wiebe found no virtuosos among his pupils. Sometimes their awkward efforts frayed his nerves. Then, in the midst of a lesson, he would stride onto his front porch, holding his hands over his ears and wailing, "Ach! Ach!" To anyone passing the house he would smile and say,*

*“But they will learn! The little ones will learn!” Turning he’d march resolutely back to his parlor and resume the lesson.*

Not long after arriving in Springfield, Wiebe presented himself at Milton Bradley’s door and taking a thick manuscript from under his arm he said, *Mr. Bradley, here is a book you will like to publish. It is about the kindergarten.* Wiebe was in no way deterred by Bradley’s prompt response that, *I’m not in the publishing business.* He countered with, *But this book is different. It is about a most wonderful man and his most wonderful idea.* Wiebe went on to tell Bradley about Froebel, about his philosophy, his methods, and his materials. Bradley did not immediately accept the manuscript which Wiebe tried to give him, but did agree to hear more about Froebel.

Wiebe described Froebel’s use of objects, how they were called “gifts” and were used as play materials to help children think about and express certain ideas. He described the six balls, each of a different color, that constituted the first gift, and the wooden sphere, cube, and cylinder that formed the second gift. He went on to describe several of the other gifts, emphasizing that they were made to show geometric relationships, to provide a variety of ways to teach form, shape, number, measurement, counting, and so on. After enumerating some of the specifics of Froebel’s methods and materials, Wiebe excitedly asked, *Do you understand the implications and significance of this?*

At the end of Wiebe’s presentation, Bradley agreed to keep the manuscript and to read it; however, he hastened to emphasize again that he was not in the publishing business. Actually, Bradley was more interested in what Wiebe had to say than he let on. What he heard Wiebe describing connected with his own childhood experiences.



As a young child growing up in Maine, life was difficult for the Bradley family. While an industrious man, his father had difficulty providing more than the bare necessities for his wife and only child, Milton. However, the lack of money did not prevent the family from enjoying a rich family life. Milton could recall his many conversations with his parents and their belief in feeding the mind as well as the body. There were books and there were games and there were *things*. Wiebe’s description of Froebel’s gifts reminded Bradley of how important and significant those games and things had been to his learning. He believed that what his parents had helped him come to know and understand through example and conversation was that learning should accompany pleasure as well as pleasure accompany learning. His biographer describes a specific instance of this, one that apparently Bradley enjoyed telling.

*He was six years old and writhing over the meaningless abstraction of first grade numerals one evening when Lewis Bradley came from the kitchen with six red apples which he placed on the table in front of Milton. “Count them,” he told his son. Milton could count to six all right; what puzzled him was why four and two should make six, as his teacher had stated.*

*Lewis Bradley removed two apples from the table and told Milton to count the remainder. Then he put back the two apples and told him to count again. Suddenly the abstract mystery of rudimentary arithmetic was solved for him. A numeral was merely a symbol for things. But it was much easier and far more pleasurable to learn with things like these bright red apples which were so pleasant to see and touch. Acquiring knowledge could be fun and it held rewards, as when his father gave him one of the apples to eat because he had learned so quickly.* (Shea)

Over the weeks that followed his presentation to Bradley of the manuscript, Wiebe made a regular pest of himself, refusing to take Bradley’s no as being final. One day Wiebe, in a state of excitement, hurried up to Bradley to tell him about a lecture to be given the follow-

ing week at a local school by a Miss Peabody, the head of a well-known kindergarten in Boston. Miss Peabody had only a few years earlier studied in Germany under the guidance of some of Froebel's disciples. Wiebe had studied under some of these same people. In fact, Wiebe had spent time studying under the direction of Froebel's widow who continued his work after his death.

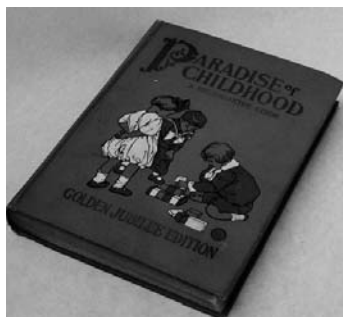
Miss Peabody's presentation made a deep impression on Bradley. She described how children responded to Froebel's methods, she described their enjoyment of learning. She argued for the need for more schools like her school in Boston where Froebel's philosophy and methods would be put into practice. Years later Bradley wrote about his impressions of that evening.

*Miss Peabody's talk carried conviction to my mind and heart, because there was something within me which sprang out to give her words welcome. The fact is, the teaching was not new. Froebel's doctrine is as old as the world. It is something innate in us all, but Froebel gave it a name, and he is justly called a philosopher. Let us not, however, exalt him by robbing human nature of something it has gained through ages of experience and teaching.*

*It is because men and women all over the world have something of the kindergarten spirit, and are unconsciously training their children in part by Froebelian methods, that the system finds such universal acceptance wherever it is taught.*

*To that evening's awakening I attribute all that I have ever done for the kindergarten, and all that the kindergarten has ever done for me.* (Shea)

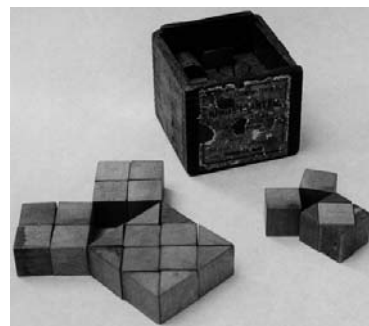
The next morning, it was Bradley who called on Wiebe. He said, *Professor Wiebe, I'd like to have your manuscript. I'm going to publish it.* Within a few months the Milton Bradley Company published its first book, *Paradise of Childhood, a Practical Guide to Kindergartners*



by Edward Wiebe. The book received a bronze medal and honorable mention at the Philadelphia Exposition of 1876.

Not only did Bradley publish the book, but he began the manufacture and distribution of the various gifts along with additional kindergarten materials. Sales of the book and materials were at first very slow and it was unclear whether the decision to pursue this enterprise of theirs was such a good idea. However, Bradley was by then so devoted to the idea of the kindergarten and the production of kindergarten materials that instead of being discouraged and giving up on the idea, he seemed energized by the challenge and the possibilities of changing the patterns of American education. Always at the forefront of his thinking about education was his belief, imbued in him by his parents, that pleasure should accompany learning. Over the next 25 years Bradley was a frequent speaker at teacher conferences across the country, always enthusiastically promoting the idea of the kindergarten, always promoting the kindergarten materials produced by the Milton Bradley Company.

Among those who attended the Philadelphia Centennial Exposition of 1876 was the mother of Frank Lloyd Wright, the renowned 20th century architect. In his autobiography, Wright noted that his mother discovered the gifts for the first time at the Exposition. He describes sitting at his "little kindergarten table" over a period of several years engaged in activities with the various Froebel gifts. Of this experience he says, *Eventually I was to construct designs in other mediums. But the smooth cardboard triangles and maple-wood blocks were the most important. All are in my fingers to this day.*



In describing the value of the experiences provided by the Froebel gifts, Thomas Banchoff (President of the

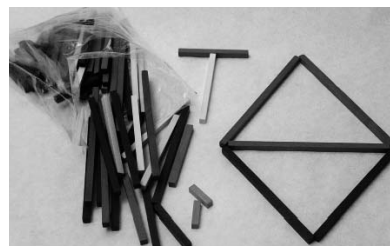
Mathematical Association of America) says, *The important thing was to introduce students to forms that they could apprehend and to encourage them to observe and recognize those forms in all of their experiences. In this way they could foster the facility of visualization, so important in applying mathematics to both scientific and artistic pursuits.*

He goes on to say, *Froebel began with objects from the most concrete part of mathematics: balls, cubes, and cylinders. He proceeded to a higher level of abstraction by presenting children with trays covered by patterns of tiles. Then, he moved further into abstraction by introducing collections of sticks of varying lengths, to be placed in designs that would ultimately be related to number patterns. ... We can recognize some of Froebel's legacy in materials that we find in today's kindergarten classrooms. There we still have blocks for stacking and tiles for creating patterns on tabletops. Too often, however, these "toys" are left behind when children progress into the serious world of elementary school.*

It seems clear that Froebel's legacy as it found its way to America would have been greatly diminished had it not been for the collaboration between Wiebe and Bradley in the late 1860s and Bradley's subsequent vigorous promotion of the kindergarten materials that resulted from that collaboration.

Some of the Froebel kindergarten gifts that were manufactured and sold by the Milton Bradley Company well into this century are pictured on these pages. While they have become antiques and command a fairly high price, their potential usefulness as a way to embody a variety of geometric concepts and relationships is undiminished.

The next article in this series will explore the eighth of the Froebel gifts. These are sticks that come in six lengths from one inch to six inches. Some of the activities Wiebe wrote about in *Paradise of Childhood* as well as other activities that have been suggested by present-day teachers who have had some experience with the materials will be described.



## References and Related Reading

Banchoff, Thomas. "Dimension." In Steen, Lynne Arthur (Ed.): *On the Shoulders of Giants*. National Academy Press. Washington, DC. 1990.

Blake, Henry W. "Life of Friedrich Froebel." In Bradley, Milton (Ed.): *Paradise of Childhood*. Milton Bradley Co. Springfield, MA. 1923.

Brosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

Shea, James J. *It's All in the Game*. G.P. Putnam's Sons. New York. 1960.

# Geometry

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## *Froebel's Gifts*

*by Richard Thiessen*

**I** *magine a group of surgeons and a group of teachers in the year 1875 on an expedition to the north pole. Caught suddenly in a severe storm, their bodies are quickly frozen and covered with many feet of snow and ice. In the year 2000, their bodies are discovered. As their bodies are allowed to thaw, each of the people awakens. They recall clearly who they are, where they lived, and the work they did as either surgeons or teachers. In fact, it is as if they went to sleep only yesterday.*

After figuring out that there is a 125-year gap for which they can give no account, but feeling fortunate to be alive and well and still the same age they were when they were frozen, they are ready to get back to their respective occupations of surgeons and teachers.

Except that the human body has not changed, there would probably be little that would be even remotely familiar to the surgeons as they tour a modern surgical suite and observe the equipment and the techniques being employed. On the other hand, the teachers upon entering a classroom for the first time in 125 years might be taken back by the computer and the overhead projector, but after a few minutes of observation of the classroom environment, they would probably feel quite comfortable with what is going on. In fact, they might very well question whether the methods and materials they observe are better or worse than those they employed 125 years earlier. With a couple of days to get used to the new environment, they might well be ready to pick up where they left off.

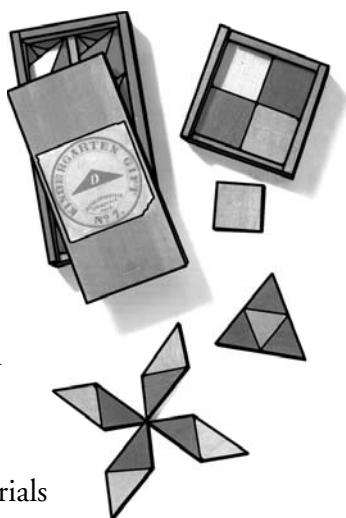
Admittedly the story is somewhat unfair. Certainly there is much about being a surgeon and doing surgery that is very different from that

of being a teacher and doing teaching. I doubt, however, that there are many times when a surgeon, in browsing through old medical school textbooks of 100 years or more ago, finds ideas, methods, and materials that might be better used and might yield better results than those of modern surgery. As a collector and browser of old math books, I often encounter ideas that today are considered new and innovative, but which were clearly presented in textbooks and other teaching materials of 100 to 150 years ago.

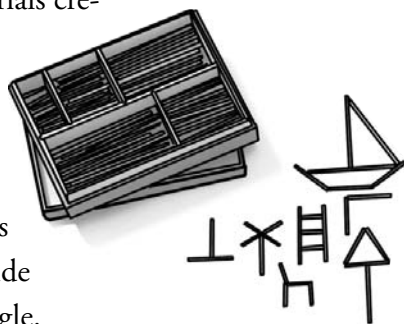
The work of Froebel in the mid-1800s is a good example of what I have just described. The multi-link cubes and the pattern blocks that can be found in many elementary classrooms today are reminiscent of several of the gifts that he created for the kindergarten. However, what he attempted to achieve with his materials was generally more sophisticated than what it appears is being achieved with the corresponding materials today. Froebel created an extensive set of inter-related materials designed to promote the growth of specific concepts and relationships along with an emphasis upon the acquisition of appropriate language. He designed activities and guides for teachers to accompany the materials. These were

based upon his understanding of how children learn and how they acquire understanding and the ability to communicate that understanding of early concepts of geometry and number.

For example, Froebel's *tablets*, Gift No. 7, are quite similar to the pattern blocks of today but they differ in some important ways. To the extent that pattern blocks are used to help children identify and classify shapes, there is little variety of shape. The only triangle in a set of these blocks is equilateral. Froebel's tablets included equilateral triangles, plus right scalene and obtuse isosceles triangles. Moreover, the tablets were part of a sequence of materials and activities that provided for a progression from three dimensions to two dimensions to one dimension to thinking about a point that has dimension zero.



The tablets are followed by Gift No. 8 which is a set of sticks of different lengths used to create a variety of shapes including the shapes of the tablets. The sticks which are one dimensional are used to create two-dimensional shapes. For example, consider how many different triangles might be constructed using the sticks. These materials create the opportunity to focus on a triangle as a simple closed curve which is really a boundary that encloses a region called the inside or interior of the triangle.



The first two articles in this series briefly tell the story of Froebel, as well as the stories of Edward Wiebe and Milton Bradley who were instrumental in

the spread of Froebel's ideas and materials in America. Three other American authors writing about Froebel's ideas and the gifts in the late 19th century were Grace Fulmer, a professor of kindergarten education at Columbia University, and two leaders in the kindergarten movement—Kate Douglas Wiggin and Nora Smith. The writing of Froebel, along with that of the other authors just mentioned, will serve as sources of information about Froebel's gifts and the philosophy and understanding of how children learn that dictated how the gifts were used and the sequence in which they were presented. Each gift and how Froebel and his followers envisioned that it would be used will be described beginning in the remainder of this article and continuing through the next two articles of this series.

The first of Froebel's gifts held for him a unique place among the 20 geometric gifts that he developed. Gift No. 1 was a set of six balls—typically these were small pliable, wool balls with strings attached. They came in six colors: red, blue, yellow, violet, green, and orange. While for subsequent gifts the activities surrounding them were designed for children in



an age range from perhaps three to eight years of age, the first gift was designed to be used with infants as a first toy. Froebel wrote extensively about how mothers could use this gift with their infants over the first years of life and how important the resulting learning would be for both the physical and mental development of the child. Froebel said of the first gift, *We must remember that to the young child, the activity of an object is more pleasing than its qualities, and we should therefore devise a series of games with the fascinating plaything which will lead the child to learn these qualities by practical experience.* (Froebel, p. 25)

In writing about the first gift Kate Wiggin said, *Froebel chose the ball as the first gift because it is the simplest shape, and the one from which all others may subsequently be derived; the shape most easily grasped by the hand as well as by the mind. It is an object which attracts by its pleasing color, and one which, viewed from all directions, ever makes the same impression.* (Wiggin, p. 6) She goes on to quote Conrad Diehl who, in commenting on the first gift said, *Color is the first sensation of which an infant is capable. With the first ray of light that enters the retina of the eye, the presence of color forces itself on the mind. When light is present, color is present. The first impression which the eye receives of an object is its color, its form is revealed by the action of light upon its surfaces.* (Wiggin, p. 8)

While Froebel placed a great deal of emphasis on the use of the first gift with the very young, other writers tended to focus on the use of the gifts with children of at least age three or four. As a consequence, these authors generally had less to say about the first gift than they did about the other gifts. In his discussion of the use of the first gift with kindergarten children, Wiebe focused on the teaching of concepts of color. He suggested that the teacher have children say things like: *My ball is green like a leaf; My ball is yellow like an lemon; Mine is red like an apple.* (Wiebe p. 14) Such activity might be followed by having children find other objects having the same color as one of the balls. Wiggin focused with kindergarten children on concepts of motion, direction, and position. About motion she said, *The ball may be made a starting point in giving the child an idea of various simple facts about objects in general, and in illustrating in movements the many terms with which we wish him to become familiar. The meaning of the terms to swing, hop, jump, roll, spring, run away, come back, fall, draw, bounce, and push may be taught by a like movement of the ball, urging the child to give his own interpretation of the motions in words. All of the children may then make their balls hop, spring, roll, or swing at the same time, accompanying the*

*movements by appropriate rhymes.* (Wiggin, p. 24) She went on to describe the use of the first gift in helping kindergarten children acquire concepts of: right/left, here/there, up/down, near/far, over/under, and front/back.

The kindergarten gifts are designed to be a series where each successive gift is implicit in the one that comes before it. Froebel selected the ball for the starting point of his series. About the succession of gifts, Wiggins said, *As the kindergarten gifts are designed to serve as an alphabet of form, by whose use the child may learn to read all material objects, it follows that they must form an organically connected sequence, moving in logical order from an object which contains all qualities, but directly emphasizes none, to objects more specialized in nature, and therefore more definitely suggestive as to use.* (Wiggins, p.8)

It is my purpose in describing each gift to give the description and use of that gift from the point of view of Froebel and others like Wiebe and Wiggins who knew his work well and attempted to faithfully interpret it for subsequent generations of kindergarten teachers. I have attempted to do that in this article with Gift No. 1. The next two articles will be devoted to similarly describing the remaining gifts. Later in this series we will come back to at least a selection of these gifts to think about how they might be developed and used with primary children today.

### References and Related Reading

Brosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

Froebel, Friedrich. *Pedagogics of the Kindergarten*. D. Appleton and Company. New York. 1909.

Fulmer, Grace. *The Use of the Kindergarten Gifts*. Houghton Mifflin Company. Boston. 1918.

Wiebe, Edward. *Paradise of Childhood*. Milton Bradley Company, Springfield, MA. 1923.

Wiggins, Kate and Nora Archibald Smith. *Froebel's Gifts*. Houghton Mifflin Company. Boston. 1896.

# Geometry

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## *Froebel's Gifts—Two, Three, and Four*

*by Richard Thiessen*

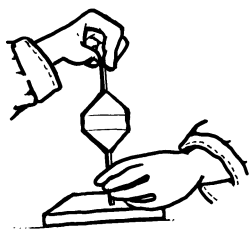
**I**n this series of articles we are exploring the work, philosophy, and beliefs of Friedrich Froebel, the originator of the kindergarten. In particular, we are exploring the geometric gifts and activities associated with them that are at the heart of his kindergarten curriculum. The last article ended with a description of the first of Froebel's geometric gifts. This article will continue with descriptions of gifts two through four. Using the references listed at the end of this article, I have tried to describe some of what Froebel and his followers envisioned as the purpose for each of the gifts. To give you a flavor of Froebel's thinking, I will liberally quote from his writing as well as that of others who have used and written about the gifts. Articles later in this series will examine how these gifts might be incorporated into activities that can be used to help primary children of the 21st century acquire concepts of shape and number.

**Gift Two** is at once the one most recognizable and the one that has created the most discussion as to how it should be used with children. As I have said in earlier articles, I first encountered Froebel's ideas about geometry in a wonderful book, *On the Shoulders of Giants*, edited by Lynne Steen. Pictured in that book was an antique version of Gift Two that had probably been constructed by a mid-19th century woodworker and which now is a museum piece. I was so intrigued by the picture in the book and the accompanying description of it and how it was used that within a couple of weeks I had built my own version. Later I discovered that Milton Bradley had built this gift along with all of the other Froebel gifts during the last quarter of the 19th and well into the 20th century. As I have probably mentioned in earlier articles, I now collect the antique Froebel gifts that were built and sold by the Milton Bradley Company. Pictured here is a version of Gift Two that is part of my collection.



The second gift as envisioned by Froebel and constructed by the woodworkers whom he employed for this purpose, consisted of a wooden sphere, a cube, and a cylinder. The sphere had a diameter of two inches, an edge of the cube had a length of two inches, and for the cylinder, the height and diameter were both two inches. Each of these shapes had hooks attached which were located strategically so that a piece of string could be looped through the hook. When the two ends of the string were held by forefinger and thumb of one hand, the child could use the other hand to repeatedly turn the shape, in this way winding the string. When the shape was

released, it would spin as the string was unwinding, allowing the student to observe yet other shapes being swept out by the spinning sphere, cube, or cylinder.



Of the cube and the sphere, Froebel said, *The sphere and the cube as solids are in respect to their form pure opposites—that is, as they are in themselves similar bodies, so are they externally opposite; thus they are opposite yet alike.* (Froebel, p. 68) Froebel believed that because of this oppositeness and also the likeness, the two objects belonged together in the child’s play.

The three distinct forms included in this gift become a basis for the recognition of different forms in the child’s environment. Just as the six colored balls of the first gift provided the opportunity for discrimination through the sense of sight, the second gift provides the additional opportunity for discrimination through the sense of touch. (Fulmer, p. 42)

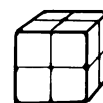
In comparing the sphere and the cube, the child finds that the sphere has one round face, while the cube has many faces. She further notices that the cube has edges and corners, which the ball does not; the ball gives the idea of motion and the cube of rest. Older children can be asked to count the number of faces, corners and edges of the cube. The cube can be placed in a variety of positions in front of the child and be asked about the number of faces of the cube that can be seen. (Wiebe, p. 24)

Comparing the cylinder to the sphere and the cube, the child should notice that it will roll like the ball because it has one round surface; it will stand or rest like the cube because it also has flat faces. (Wiebe, p. 25)

Milton Bradley, in writing about this gift and describing some of the ways in which children might

use the three objects to represent different things in their environment, said that, *In using the same form to represent different things in play, do not fear that there will be any incongruity, provided the suggestion comes from the children, and the objects symbolized are closely related in thought, for the child’s imagination is so free that he can clothe and re-clothe the same form with new life. The sense impressions which come from tracing resemblance and differences, experimenting and handling, will give a familiarity with the forms and their relation to each other, which no abstract lesson on surfaces, edges, and corners could afford. . . . No such wealth of resources to cultivate imagination and inspire confidence is found in any other gift as in this, which was an especial favorite with Froebel, and is so invaluable that no kindergarten teacher who has once shared the delight of the children in this gift for one year in the kindergarten course, will ever be willing to do without a box for each child.*

**Gift Three** is simply a set of one-inch cubes that can be stacked-up to form a two-inch cube. The cubes are presented to children in a two-inch box so that the box can be turned upside down and then be removed to reveal the two-inch stack of cubes. Children are initially encouraged to take the stack apart and then attempt to put it back as it was or in some other arrangement. Whereas the first two gifts involved objects that were each in a sense a whole that couldn’t be taken apart, this gift is the first of what he called the building gifts where the emphasis is on taking apart and putting together, on seeing the parts in relation to the whole. He called this gift, “the children’s delight.”

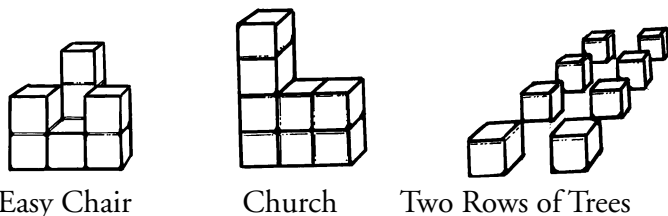


In describing this gift, Froebel says, *The principal cube appears separated by the mentioned division in this play into eight equal cubes. The child thus distinguishes here as a given fact, and without any words (purely as the perception of an object), a whole and a part, for*

each component cube is a part of the principal cube. The component cubes have the same form as the principal cube; thus what the principal cube shows once in respect to its form, the component cubes show together as often and as repeatedly as there are cubes. ... He thus again distinguishes purely as a perceptible fact the size from the form, for each component cube shares indeed the cubical form of the principal cube, but not its size. ... Therefore, by this simple play the fundamental perceptions, whole and part, form and size, are made clear by comparison and contrast, as well as deeply impressed by repetition. The child further perceives, as a fact, position, and what is yet more important, arrangement; for before him is shown an above and below, an over and under, a behind and before, etc. Hence, one upon the other, one behind the other, and one beside the other, etc.

The third gift satisfies the growing desire for independent activity, for the exercise of the child's own power of analysis and synthesis, of taking apart and putting together. (Wiggin, p. 59)

As an early activity with the cubes of Gift Three, Froebel asked children in their play to stack them in a variety of ways to represent things in their surroundings. For example, they might be stacked to represent a chair or a table. In many ways these activities remind me of some of the more contemporary activities with tangram pieces where children are asked to put the pieces together to form a variety of shapes representing animals or people or other objects that they can imagine.



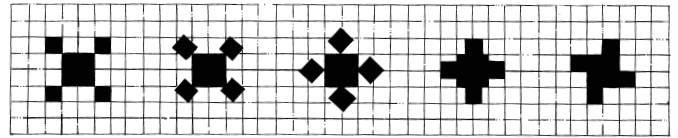
Easy Chair

Church

Two Rows of Trees

With the cubes Froebel also encouraged the use of a mat or table top that was grided in one-inch squares so that the cubes could be placed in more orderly arrangements. Using the grid as a background, children were shown how to make various symmetric

arrangements of the cubes. He believed that children would not only begin to recognize the symmetry in the arrangements, but would also see beauty in them.



Edward Wiebe in *Paradise of Childhood* gives a demonstration of how the eight cubes might be used with children to help them think about the parts of the whole in different ways. The following dialogue is taken from that book.

The children have their cube of eight before them on the table and the teacher has one as well.



The teacher, lifting the upper half asks, *Did I take the whole of my cube in my hand or did I leave some of it on the table?*

Stu: *You left some on the table.*

Tch: *Do I hold in my hand more of my cube than I left on the table or are both parts alike?*

Stu: *Both are alike.*

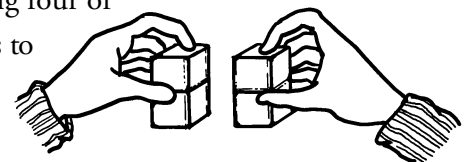
Tch: *If things are alike, we call them equal. So I divided my cube into two equal parts, and each of these equal parts I call a half. Where are the two halves of my cube?*

Stu: *One is in your hand; the other is on the table.*

Tch: *So I have two half cubes. I will now place the half which I have in my hand upon the half standing on the table. What have I now?*

Stu: *A whole cube.*

The teacher, then separating the cube again into halves, by drawing four of the smaller cubes to the right and four to the left asks:

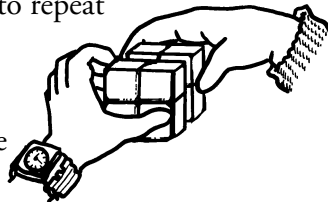


*What have I now before me?*

Stu: *Two half cubes.*

Tch: *Before, I had an upper and a lower half. Now, I have a right and a left half. Putting together the halves again I have once more the whole.*

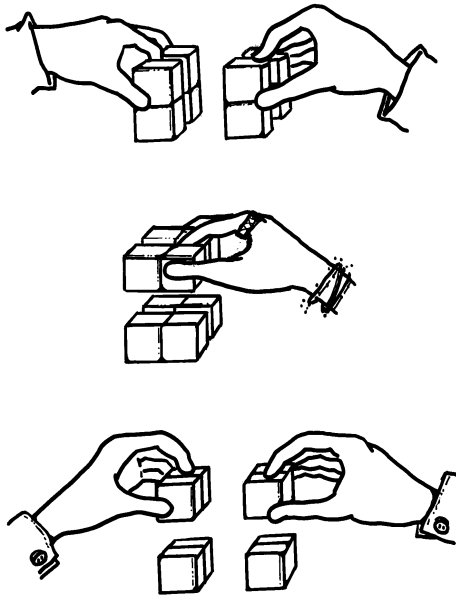
The students are taught to repeat as follows while the teacher divides and puts together in both ways, and also does the same for the front four and the back four cubes.



Tch: *One whole—two halves.*

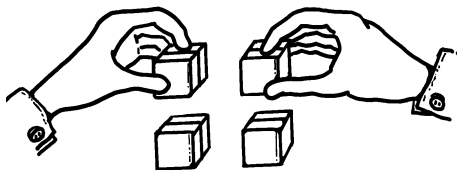
Tch: *Two halves—one whole.*

Going back now to the three ways in which the cube was divided into halves, each of the corresponding halves are now themselves divided into halves. The children are asked to repeat these activities of dividing the cube in these various ways.



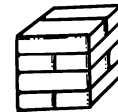
Tch and Stu: *One whole—two halves. One half—two fourths. Two fourths—one half. Two halves—one whole.*

After these processes are fully explained and the principles well understood by the students, they are to try their hand at dividing the cube.



A final extension suggested by Wiebe in this dialogue was that for some children it might be possible to make a final step of having them separate the cube into eighths.

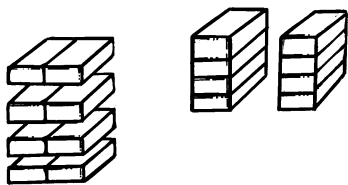
**Gift Four** is again a two-inch cube that has been subdivided into eight equal parts. This time the parts are rectangular solids that are  $\frac{1}{2}$  by 1 by 2. Froebel introduced this gift to children in much the same way as the third gift. The box containing the eight pieces is placed lid down and then the lid is pulled out and the box lifted up revealing the two-inch cube. Each of the cube gifts is initially presented to the child in this way because Froebel believed that the child should begin her play with the gift in such a way that it will be perceived as a whole rather than many parts. Moreover, he believed that the initial way in which the pieces are arranged should be such that the characteristics of each of the pieces will be discernible. He felt that by seeing the shapes of the various pieces still arranged as a whole, the child will, before ever taking it apart, begin to think and plan how the pieces might be used in other ways.



While the children are helped to notice that the whole cube has the same size as the previous one, when taken apart it is obvious to them that the pieces are very different. They are encouraged to compare the pieces of this gift with those of the third gift—to note how they are different and how they are alike. In the previous gift the pieces had the same shape as the whole cube; in this case the pieces have a shape very different from that of the whole. They may be able to notice that the number of the pieces is the same, and they may also be able to determine that in some way the pieces are alike since each set can be put back together to form an identical whole.

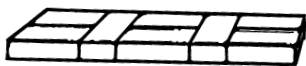
While the third gift involved cutting the two-inch cube in half in each of its three dimensions where the plane of each cut is perpendicular to the planes of the

other two cuts, this gift involves cuts in only two of the dimensions. There is one vertical cut and three horizontal cuts. The children can notice these cuts by taking the cube apart layer by layer to reveal the four layers which result from the horizontal cuts and can separate the cube into two stacks of four of the oblong pieces to reveal the vertical cut.



One of Froebel's objectives for the cube gifts was to focus attention on the three mutually perpendicular dimensions of any solid shape. He was concerned that in the case of the cube pieces these dimensions were not as distinguishable since the length of the cube in each dimension was the same. He saw in the oblong blocks the opportunity to call attention to the dimensions since the length measure is different in each dimension.

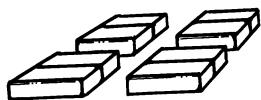
Froebel also saw in the fourth gift the opportunity for children to produce a richer variety of shapes and figures that might correspond with forms of objects in their surroundings. These pieces could be laid flat to produce surface forms or could be set on end to produce a wall or enclosure. He felt that it was important that whatever objects children produced with the pieces that they name the objects by connecting them to something familiar in their environment. For example, they might be put together to represent a house or a table.



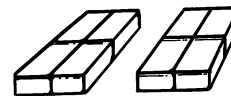
**Part of a Floor, or Top of a Table**



**A Long Garden Wall**



**Four Small Boards**



**Two Large Boards**

In addition to producing a variety of objects through free play, Froebel also asked children to produce various forms that he would show them. Wiebe, in *Paradise of Childhood*, pictures 50 of these forms that were apparently designed by Froebel. While many of them could be found in free play, these particular forms were suggested to the children so that they might copy them.

Froebel believed strongly that the geometric gifts formed a sequence such that each new gift was suggested by the previous gift and built upon it. While Gifts Three and Four involved looking at dividing the two-inch cube in different ways, the cuts were always parallel to the faces. The next gift in the series provides yet another extension, one in which not only is the whole cube cut along planes parallel to the faces, but some of the resulting pieces are themselves cut into halves and fourths. We will continue with Gifts Five and Six in the next article of this series.

### References and Related Reading

Brosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

Froebel, Friedrich. *Pedagogics of the Kindergarten*. D. Appleton and Company. New York. 1909.

Fulmer, Grace. *The Use of the Kindergarten Gifts*. Houghton Mifflin Company. Boston. 1918.

Wiebe, Edward. *Paradise of Childhood*. Milton Bradley Company. Springfield, MA. 1923.

Wiggins, Kate and Nora Archibald Smith. *Froebel's Gifts*. Houghton Mifflin Company. Boston. 1896.

# Geometry

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## *Froebel's Gifts—Five and Six*

*by Richard Thiessen*

**W**e continue in this article the story of Froebel's geometric gifts and some of the ways in which he and his followers proposed that they be used. Froebel believed that the desire for activity was innate in the child and that it was therefore fundamental that all education should begin with finding ways to satisfy and further develop this innate desire. It was this belief that gave rise to the development of the gifts and the ways in which he proposed that they be used with children. Moreover, he believed that the sequence of gifts was so arranged that each successive gift was a logical consequence of those preceding it.

We concluded the previous article with Gift Four and so begin here with Gift Five. Gifts Three and Four begin with the subdivision of a two-inch cube into eight equal pieces. Gifts Five and Six begin with a three-inch cube subdivided into 27 equal pieces, with some of these pieces being further divided.

It is interesting to note that Froebel separated the activities associated with a particular gift into three different types.

- The initial activities with a gift generally involved using the pieces to construct representations of familiar things in the child's surroundings. He called activities of this kind forms of life. As children are introduced to successive gifts, the possibilities for doing this in more and more realistic ways increase.
- A second type of activity, which he introduced with each gift, was called forms of beauty. These activities involve using the pieces to form a variety of symmetric arrangements of the pieces in either two or three dimensions.
- While the creation of these symmetric arrangements was certainly geometric and mathematical, he considered a third type of

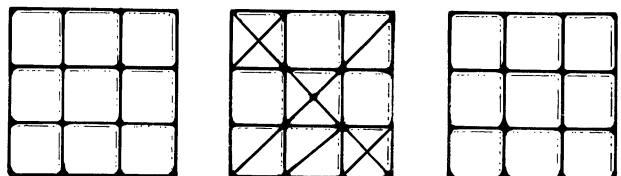
activity, which he called forms of knowledge, to be more deliberately mathematical.

It appears that both the activities associated with his forms of beauty and those that he called forms of knowledge often extend well beyond the mathematical maturity of kindergarten children.

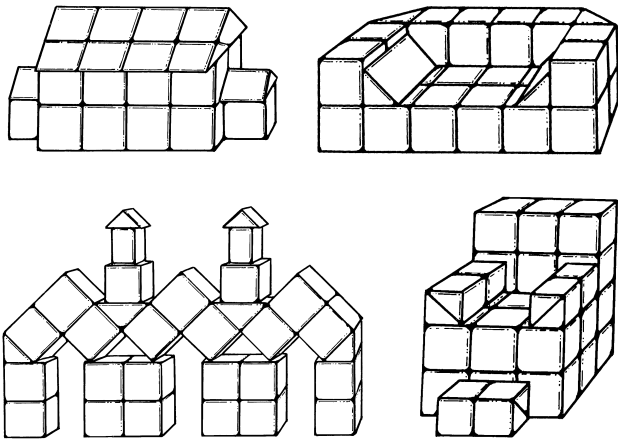
As you read, notice how these forms are woven into the activities associated with the gifts.

### **Gift Five**

Gift Five appears to be an extension of Gift Three. Gift Five consists of 27 one-inch cubes which are the result of cutting a three-inch cube along two parallel planes in each dimension. Three of the resulting cubes are then cut in half along a vertical diagonal plane and another three are cut into fourths by two intersecting diagonal planes. Froebel saw the importance of these diagonal cuts as providing a contrast to the cuts that are parallel and/or perpendicular to the faces.

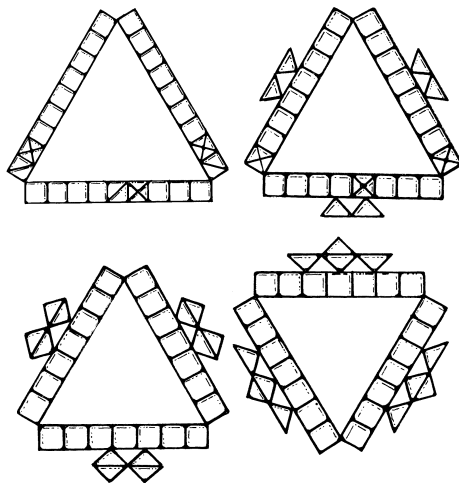


Just as Gift Four allowed for more realistic representations of familiar things than did Gift Three, so does Gift Five. With the half and quarter cubes formed by the diagonal cuts it was now possible, for example, to represent the sloping roofs of houses (*forms of life*). A few examples of such representations are pictured below.

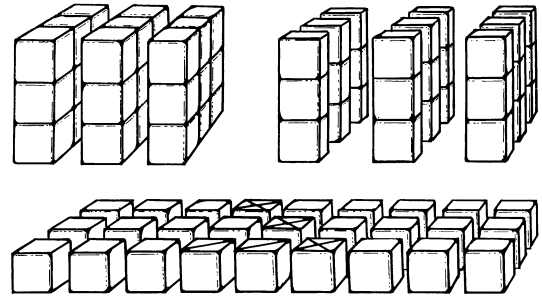


An interesting requirement that Froebel made for each activity with a given gift was that all of the pieces of the gift be used for each object that was constructed. For example, if the child were using the pieces to represent a house, all of the pieces needed to be used. If three pieces were leftover, the child was asked to find something that could be added to the structure that would use them up. Extra pieces might become a porch, a tree, or a planter.

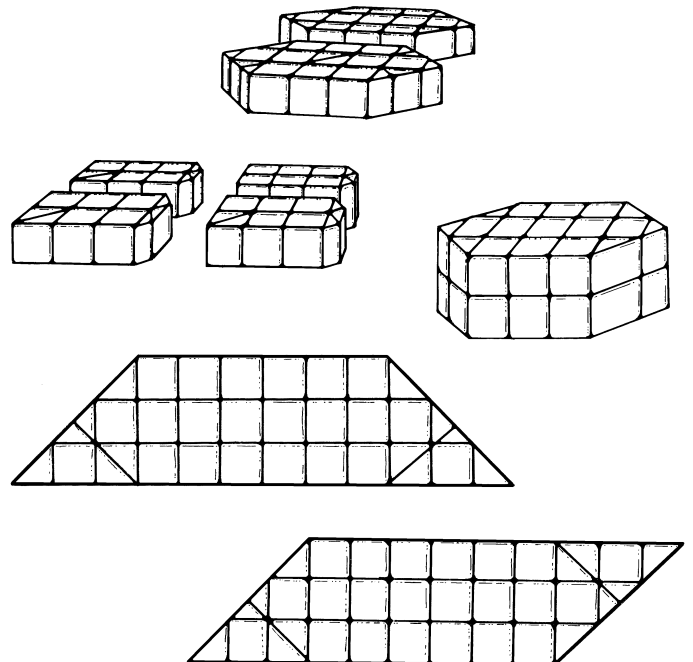
With 27 cubes and 36 total pieces, the possibilities for making symmetric arrangements of the pieces is virtually endless (*forms of beauty*). In making such arrangements, Froebel emphasized the importance of having children create successive arrangements by modification of previous arrangements. Such a sequence is pictured here.



The possibilities for representing concepts and relationships of shape and number are greatly increased with this gift and clearly the possibilities go well beyond what would be expected of a kindergarten child. Just as with the previous gifts, the early activities involve separating the  $3 \times 3 \times 3$  cube into congruent thirds in different ways, then into ninths, and finally into twenty-sevenths (*forms of knowledge*).

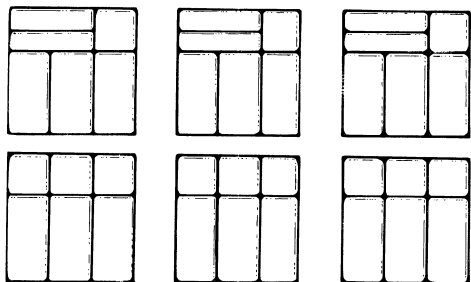


The half and quarter cubes provide the opportunity to make more interesting subdivisions into congruent pieces. For example the pieces can be separated into two, four, six, and 12 congruent arrangements. These can in turn be stacked up to form interesting geometric solids. Froebel emphasized the importance of thinking about how pieces can be put together and taken apart. Moreover the pieces can be used to form a variety of quadrilaterals each having an area of 27.

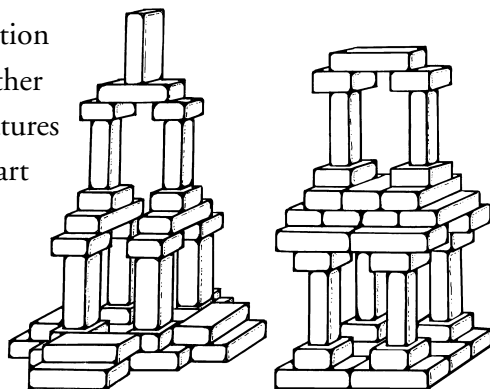


## Gift Six

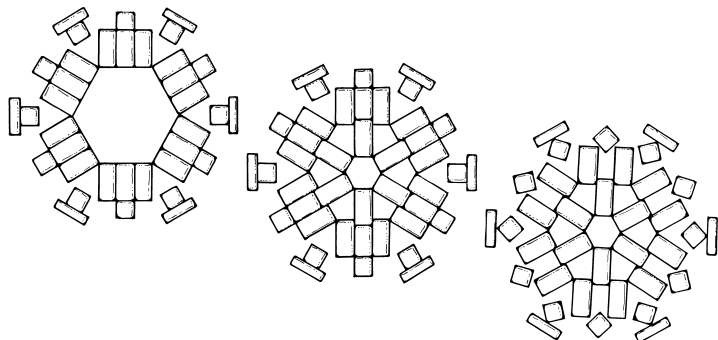
Gift Six is intimately connected to Gift Four in that the  $3 \times 3 \times 3$  is first divided into 27 oblong,  $1 \times 2 \times \frac{1}{2}$  pieces. Of these 27 pieces, six of them are in turn cut in half widthwise to form  $1 \times 1 \times \frac{1}{2}$  pieces and three of them are cut in half lengthwise to form  $\frac{1}{2} \times \frac{1}{2} \times 2$  pieces. Thus this gift too consists of 36 pieces.



The pieces of this gift can be used in much the same way as those of Gift Four to represent a variety of things in the child's surroundings. The additional pieces further allow for the construction of towers and other architectural features that might be part of the school building or of a nearby church with which the children are familiar (*forms of life*).



There are endless possibilities for creating symmetric arrangements with these pieces (*forms of beauty*). Wiebe pictures several sequences of arrangements of these pieces where each successive arrangement is so modified as to bring to mind a kaleidoscope.



Wiebe pictures many possible arrangements of the pieces of Gifts Five and Six to show both representations of things that children might observe in their surroundings and also the many ways in which the pieces might be placed in symmetric arrangements. Looking at them, I cannot help but think of the many things that Frank Lloyd Wright had to say about these gifts and the impact that his youthful experience with the gifts had on his later work as an architect.

Gifts Three through Six are sometimes referred to as the building gifts. All six of the gifts described to this point have been three-dimensional. The next two gifts which will be described in the next article of this series are one- and two-dimensional.

## References and Related Reading

Brosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

Froebel, Friedrich. *Pedagogics of the Kindergarten*. D. Appleton and Company. New York. 1909.

Fulmer, Grace. *The Use of the Kindergarten Gifts*. Houghton Mifflin Company. Boston. 1918.

Wiebe, Edward. *Paradise of Childhood*. Milton Bradley Company. Springfield, MA. 1923.

Wiggins, Kate and Nora Archibald Smith. *Froebel's Gifts*. Houghton Mifflin Company. Boston. 1986.

# Geometry

## *Froebel's Gift—Seven*

by Richard Thiessen

All mental development begins with concrete beings. The material world with its multiplicity of manifestations first attracts the senses and excites them to activity, thus causing the rudimental operations of the mental powers. Gradually—only after many processes, little defined and explained by any science as yet, have taken place—man becomes enabled to proceed to higher mental activity, from the original impressions made upon his senses by the various surroundings in the material world. (Wiebe, p.149)

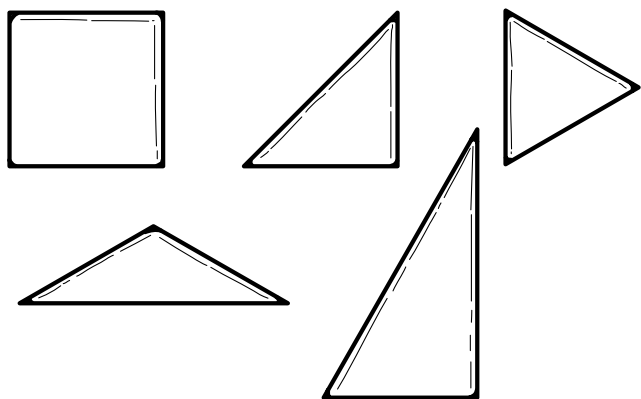
With this paragraph, Edward Wiebe introduces the seventh of Froebel's geometric gifts. Going from Gift Six to Gift Seven there is a shift from three-dimensional to two-dimensional materials and Wiebe wants to make sure that the readers and users of the gifts clearly understand the significance of this shift. While the first six gifts provided ways to make three-dimensional representations of real-world objects, Gift Seven requires that these representations be two-dimensional. While Wiebe accepted Froebel's beliefs in the importance of concrete experiences for intellectual development and for the growth of conceptual knowledge and higher level thinking, he clearly understood the need for further work in this area. It would seem that he anticipated the work of such men as Vygotsky and Piaget and the present emphasis on the construction of knowledge. Moreover, it is clear that he was confident that such work would confirm the correctness of Froebel's beliefs. In the second paragraph of his introduction to Gift Seven, Wiebe continues to focus on the transition from three to two dimensions and the transition from the concrete to the abstract.

*The earliest impressions, it is true, if often repeated, leave behind them a lasting trace on the mind. But between this attained possibility to recall once-made observations to represent the object perceived by our senses, by mental image (imagination), and the real thinking or reasoning, the real pure abstraction, there is a very long step, and nothing in our whole system of education is more worthy of consideration than the sudden and abrupt transition from a life in the concrete, to a life of more or less abstract thinking to which our children are submitted when entering school from the parental house. (Wiebe, p. 149)*

Gift Seven consists of five different two-dimensional shapes which Froebel referred to as tablets or tiles. There is a square with a side of one inch; an equilateral triangle with sides of one inch; a right isosceles triangle with legs of one inch; an obtuse isosceles triangle with angles of 30, 30, and 120, where the equal sides have lengths of one inch; and a scalene triangle with angles of 30, 60, and 90, where the short leg has a length of one inch.

In the late 1800s Milton Bradley produced Gift Seven as a set of tiles which included these

five pieces, but to which he added a one-inch circle, a one-inch semi-circle, and a rectangle having sides equal to the legs of the 30-60-90 triangle. Sets of these pieces were available in 1/8-inch thick wood in two colors or in cardstock with multiple colors.



(A page with multiple copies of these pieces appears at the end of this article. You may want to make several copies of that sheet and try out some of the things that Froebel envisioned children doing with the pieces of this gift.)

This gift is similar to the pattern blocks of our day, but with some very distinct differences. Froebel placed a great deal of emphasis on the triangle, including four different triangles in his set of tablets or tiles. A set of pattern blocks, which has several shapes not included in Gift Seven, has only one triangle, the equilateral. While the pattern blocks allow for making a variety of tilings, they tend to provide fewer opportunities to explore the kinds of geometric relationships which are possible with Froebel's tablets. As we look at each of the pieces individually, we will focus on some of the geometric concepts, which Froebel viewed as important for young children to notice and explore.

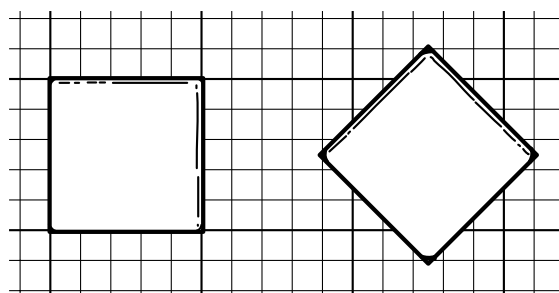
### Square

In Froebel's sequence of gifts, children would by now have had extensive experience with geometric solids. To help children explore the connection between the square and the cube, he would have them place a square on each face of a cube. While seeing that the face of the cube is a square may be obvious to the

adult, the child may well have difficulty identifying the face as a square.

The following questions are examples of the kind that Froebel suggested might be asked of children in exploring the square. Clearly, some of these questions demonstrate that these materials are designed to be used with children over a range of ages and grade levels. How many of such square faces does a cube have? How many sides does a square have? Are they alike or different? How many corners does a square have? Two sides come together at a corner to form an angle. How many angles does a square have? Are they alike or different?

Just as with the cubes in the earlier gifts, Froebel encouraged children to place the squares on a grid and see that the square pieces match the squares of the grid just as was done with the cubes of the earlier gifts. In doing this he wanted children to notice that the square could be placed so that of two adjacent sides, one was vertical and the other horizontal. Moreover, he told them that two such sides form an angle that is called a right angle. He went on to have the children place the squares on the grid in other positions so that they could notice that while the sides are no longer respectively horizontal and vertical, the angles have not changed, they are still right angles.

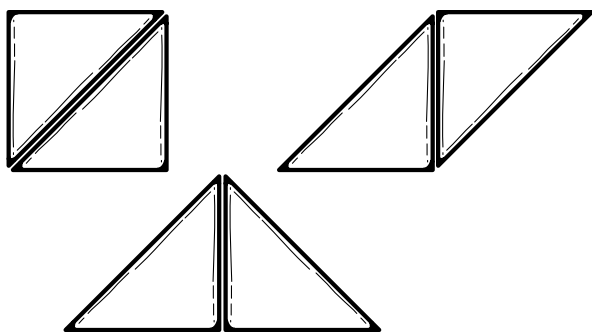


### Right Isosceles Triangle

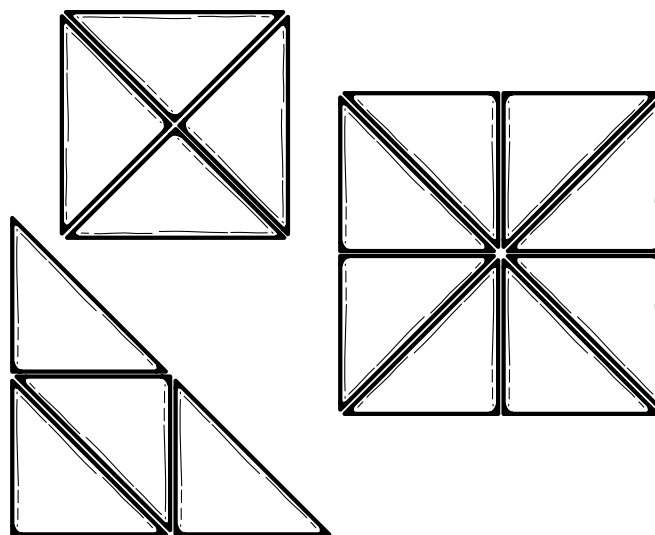
Children who have experienced the previous gifts will already have encountered the right isosceles triangle in Gift Five where some of the cubes were cut in half along a diagonal. Froebel began by presenting two of these triangles put together to form a square. In this way, children viewed this triangle as two parts of a square. The following are typical of the kinds of questions that Froebel asked children about these triangles.

Separate the two parts of the square, and look at each one separately. What do you call each of the parts? What did you call the whole? How many corners has the square? How many corners or angles has the half of the square you are looking at? Notice that one side is longer, the other two are shorter. What kind of angle do the two equal sides form? What would you call the other two angles? How do the sides run that form the other two angles? Notice that they run in such a way as to form a very sharp point. These are called acute angles. This triangle has two different kinds of angles: one right angle and two acute angles. (Wiebe, p. 150)

Having first encountered two of these triangles put together to form a square, children might be led to think about the square as cut in half along a diagonal. Now if the two triangles are put together so that one of the shorter sides of one of the triangles is matched with one of the shorter sides of the other triangle, there are two possible shapes that can result. Put together one way they form a parallelogram. With the same sides put together, but with one of the triangles flipped, the resulting shape is another right triangle. Although twice the size, it too is a right isosceles triangle.



With four of these triangles placed on a square grid, children can be challenged to create a variety of symmetric shapes. Among the shapes is a square and yet another right isosceles triangle. What are the symmetric shapes that might be created with eight triangles or 16 triangles? Is a square possible in each case? What can be said about the sides of these squares? Is a right isosceles triangle possible in each case?



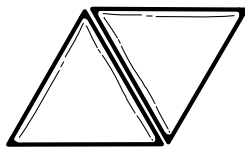
Different from what we today might do with materials like these, Froebel was careful to give children only one shape at a time and ask them to explore it at some length before presenting a new shape. With these shapes his ordering was deliberate in that he started with the square, then moved to the right isosceles triangle in such a way that he was immediately asking the children to relate the new piece to the previous one. Now you will see that as he presents children with the equilateral triangle, he again immediately relates this piece to the right isosceles triangle.

### Equilateral Triangle

*The child will naturally compare the equilateral triangle, which he now receives with the isosceles, right-angled tablet already known to him. Both have three sides, both three angles, but on close observation not only their similarities, but also their dissimilarities will become apparent. The three angles of the new triangle are all smaller than a right angle; hence they are acute angles. Moreover, the three sides are just alike; hence the name—equilateral—meaning equal sided.* (Wiebe, p. 156)

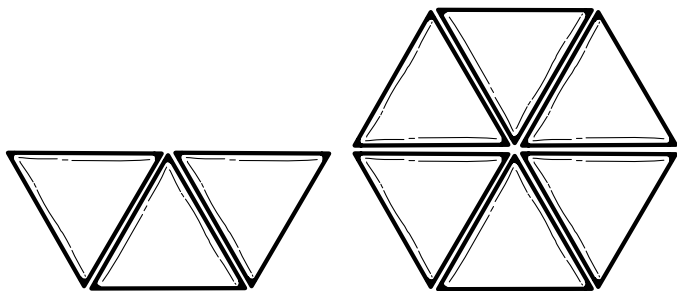
Joining two of these triangles as was done with the right isosceles, children will find that it is not possible to create either a triangle or a square. In fact, joining two of these equilateral triangles together side to side creates a shape that may well be new to the children. Investigating the rhombus, children might be asked

to compare the sides: Are they alike? Are they parallel? Examining the angles, children should find that opposite angles are the same and that one of the pairs of angles is acute and the other obtuse.



Comparing this experience with that of the square, children might be asked how a rhombus is different and how it is like a square. They should find that both have sides that have the same length and that opposite sides are parallel. Moreover, opposite angles are equal for both shapes. They are different in that all angles of the square are right angles, while no angles of this rhombus are right angles. Further careful investigation and noting how the rhombus was created may reveal that one pair of equal opposite angles are twice the size of the other pair.

Other shapes can be explored using various numbers of equilateral triangles. The trapezoid can be constructed using three of the triangles. With an adjustment to the orientation of one of the triangles, the trapezoid can be transformed into a parallelogram. Using six pieces, all of which share a common point, the pieces can be used to form a regular hexagon. How many of the triangles would be needed to create another, larger trapezoid? What is the fewest pieces needed to create another, larger triangle?

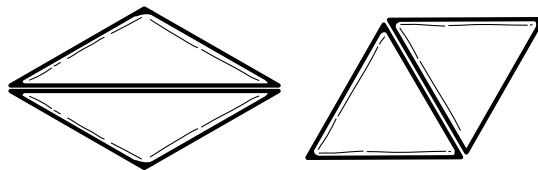


### Obtuse Isosceles Triangle

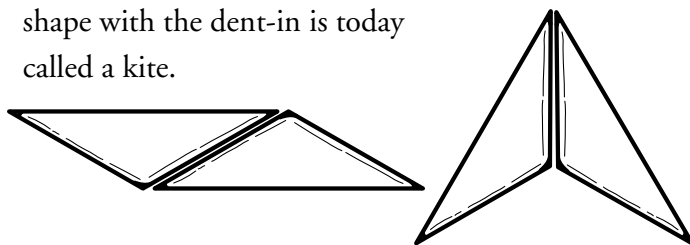
Given the experiences of examining the previous two triangles, children are by now aware of what to

look for in making comparisons. They may quickly notice that two sides of this triangle are the same and that one angle is obtuse and two angles are acute. Comparing it to the right isosceles triangle, children can be asked how they are alike and how they are different. Both have two sides the same and both have equal acute angles. Comparing the third angles it will be clear that one is larger than the other—one is obtuse and the other right. How do the third sides of the two triangles compare? Is the side opposite the obtuse angle of the one triangle longer than the side opposite the right angle of the other?

Comparing this triangle to the equilateral will reveal that there are not many ways in which the two are alike. However, if the rhombus formed by putting together two of the equilateral triangles is compared to the shape formed by joining two of obtuse isosceles triangles along the longer side, it will be immediately clear that the two shapes are the same. Children might notice that the only difference between them is that different diagonals of the rhombus separate the two pairs of triangles.



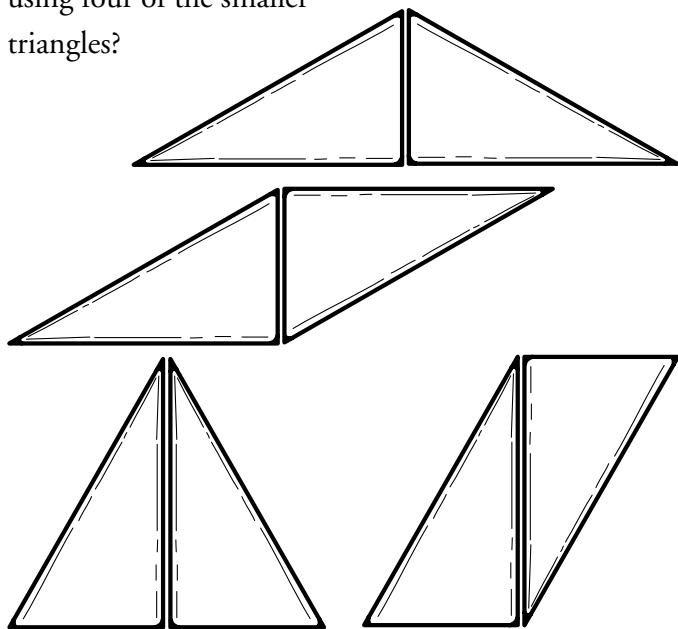
If two of these triangles are joined along one of the two equal sides, two possible shapes may result. One of these is a parallelogram; the other is a four-sided shape that has a dent-in. This shape is different from anything that the children have yet experienced. While the opposite sides of the parallelogram are equal, the sides next to each other are unequal. The shape with the dent-in has two pairs of sides next to each other that are equal, but opposite sides are unequal. Although Froebel did not use this language, the shape with the dent-in is today called a kite.



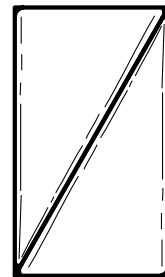
## Right Scalene Triangle

Children now encounter the fourth and final triangle of this gift. With what they know from the other triangles, they should quickly note that this triangle is a right triangle just as was the first triangle. They should further notice that no two sides of this triangle are the same. That makes it different from all of the other triangles they have explored. Moreover, they should notice that the two acute angles are different, so that not only are no two sides the same, but it is also the case that no two angles are the same.

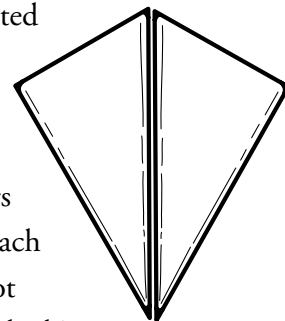
If two of these triangles are joined along the shorter of the two legs, two different shapes are possible. If the pieces are put together so that equal angles are together, the shape is an obtuse isosceles triangle. If they are joined so that unequal angles are together, the shape is a parallelogram. Similarly, when the two triangles are joined along the longer leg, a parallelogram is created if unequal angles are together at either end of the shared side. However, something very interesting happens when they are joined so that equal angles are together. Not only is the shape created a triangle, it is an equilateral triangle. It is a larger version of the equilateral triangle piece of the gift. How does this equilateral triangle compare to some of the triangles that were created earlier using several of the smaller equilateral triangles? Is it the same as the one created using four of the smaller triangles?



What shapes are created when two of these triangles are joined along the side opposite the right angle? Note that when unequal angles are together, the shape is a rectangle, where the joined sides form a diagonal. While two of the angles of the rectangle are clearly right angles because they are the right angles of the triangle, the other two angles which are created by putting together two acute angles are also right angles. Children may be able to notice that this means that the two unequal acute angles of this triangle must together form a right angle.

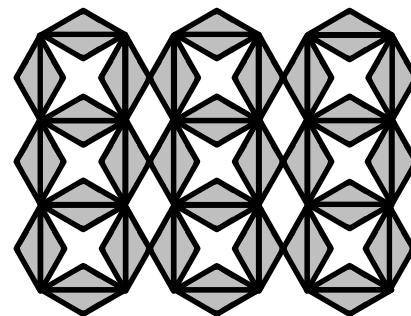


When the two triangles are joined along the side opposite the right angle so the equal angles are together, the shape created is very much like that of the shape with the dent-in created by putting together two of the obtuse isosceles triangles. While this shape doesn't have a dent-in, it does have two pairs of equal sides that are next to each other, and opposite sides are not equal. Again, this shape is called a kite.

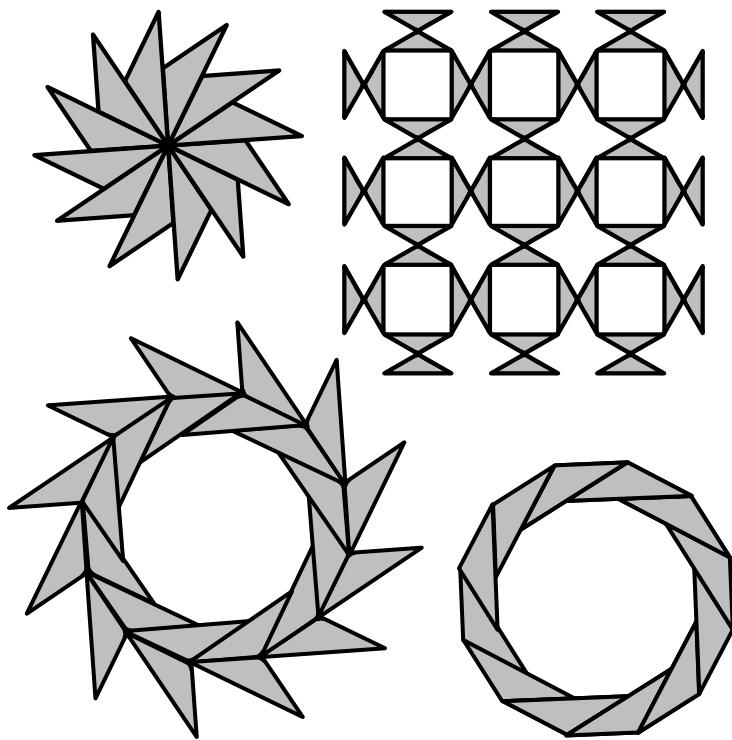


## Forms of Beauty

We have so far focused on what Froebel called *forms of knowledge*, that is, we have focused on the mathematical and geometric ideas that are embedded in this gift. There are also some tremendously interesting *forms of beauty*, e.g., symmetric shapes and mosaics that can be created with the pieces of this gift. Rather than give examples of *forms of beauty* for each of the shapes, I will simply show a few of my favorites and let you create others on your own. The two mosaics and the three symmetry shapes are created using only obtuse isosceles triangles.



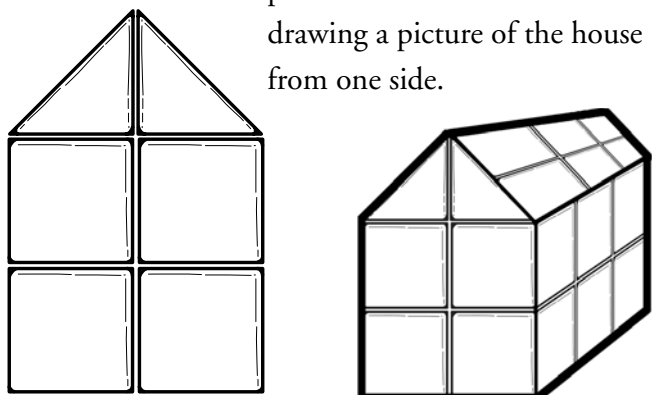
Using a variety of colors in the pieces can enhance the beauty of these creations.



### Forms of Life

As we mentioned earlier in the article, Froebel viewed the transition from three- to two-dimensional pieces as a major shift in the way in which children think about creating representations of objects in their three-dimensional world. I will give only one example to illustrate the difference between the two representations. When children used the cubes and half and quarter cubes of Gift Five to construct a representation of a house, it was possible to show all four outside walls as well as the entire roof. Using the blocks, children essentially constructed a scale model of full-sized house. To represent that house using the two-dimensional

pieces of Gift Seven is more like drawing a picture of the house from one side.



There is an amazing number of geometric concepts and relationships having to do with triangles and quadrilaterals that are embedded in the pieces and activities of this gift. The pieces of this gift make the exploration of many of those ideas at an intuitive, informal level readily available to children long before they are formalized in a middle school or high school geometry course.

We'll talk about Gift Eight in the next issue of the magazine.

### References and Related Reading

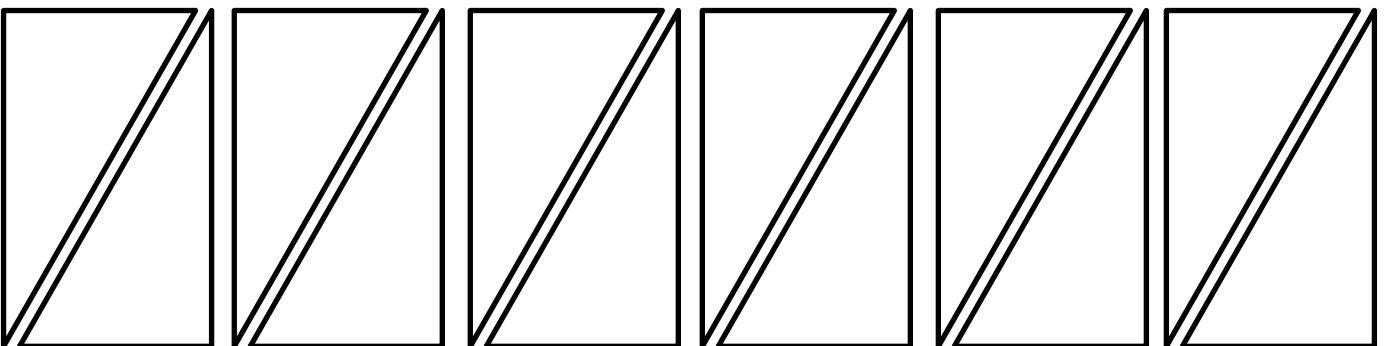
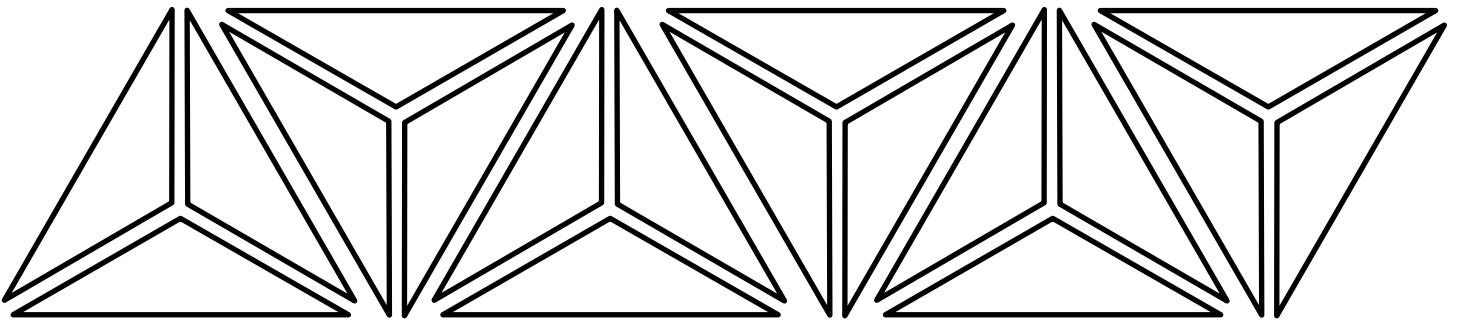
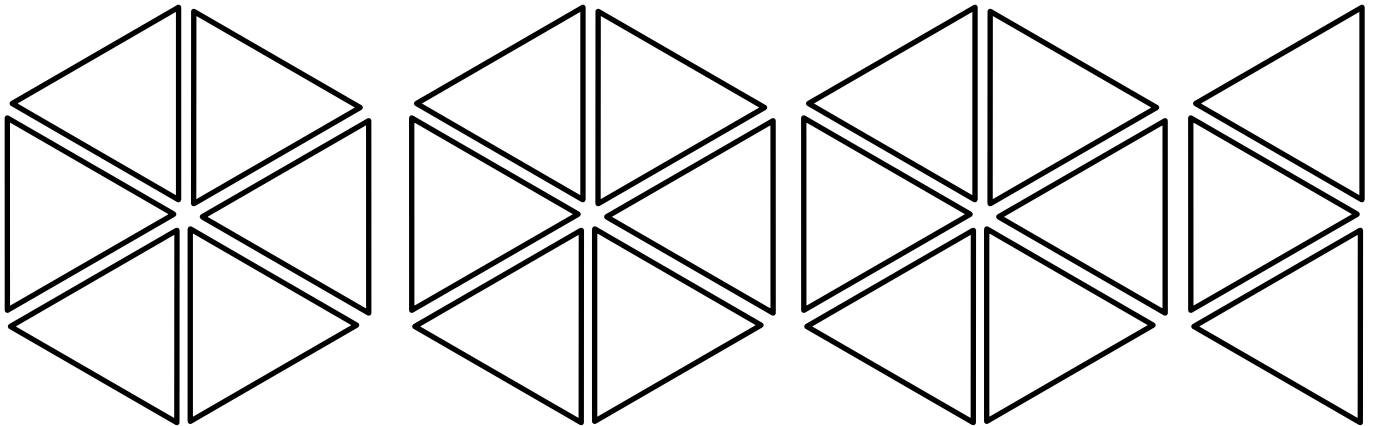
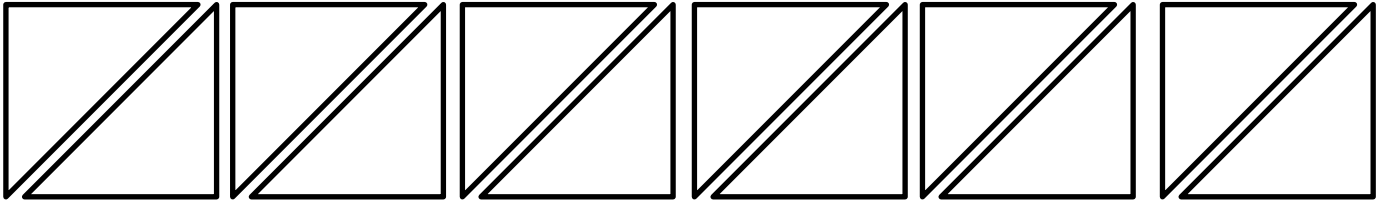
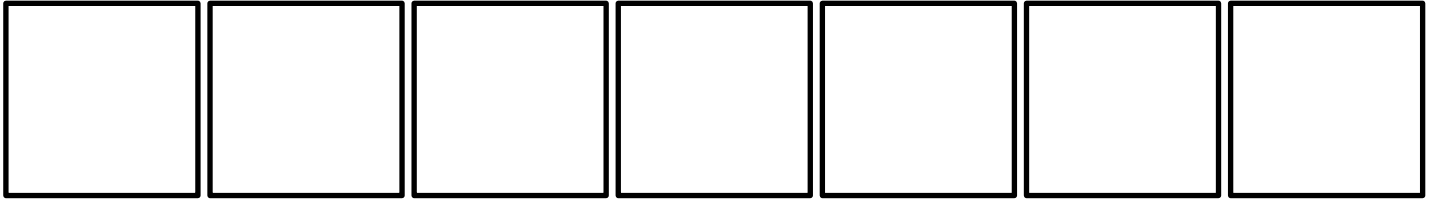
Brosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

Froebel, Friedrich. *Pedagogics of the Kindergarten*. D. Appleton and Company. New York. 1909.

Fulmer, Grace. *The Use of the Kindergarten Gifts*. Houghton Mifflin Company. Boston. 1918.

Wiebe, Edward. *Paradise of Childhood*. Milton Bradley Company. Springfield, MA. 1923.

Wiggins, Kate and Nora Archibald Smith. *Froebel's Gifts*. Houghton Mifflin Company. Boston. 1986.



# Geometry

## *Froebel's Gift—Eight*

*by Richard Thiessen*

**T***his is the last in a series of articles that has focused on the materials and activities of Froebel's first eight geometric gifts. Several nineteenth-century books written by followers of Froebel have been used as sources, and it is their description of the materials, activities, philosophy, and beliefs that we have been exploring through this series. Primary among the sources is Edward Wiebe's book, Paradise of Childhood, first published in 1869 by the Milton Bradley Company. An earlier article of this series was devoted to the story of Wiebe and his relationship with Milton Bradley that made the publication of his book a reality.*

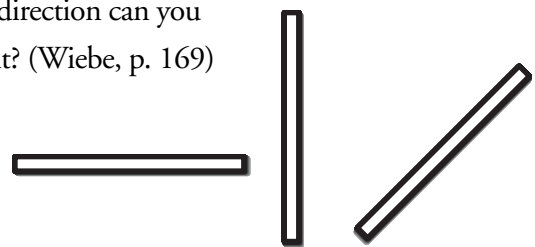
Sticks for Laying of Figures is the subtitle that Wiebe uses for the section of his book in which he describes Gift Eight. This gift consists of sticks of lengths from one to five or six inches. Manufactured by Milton Bradley, they were available painted or unpainted, where each length was painted a different color. Just as the tablets of Gift Seven are somewhat like the pattern blocks of today, the painted sticks of Gift Eight are somewhat like Cuisenaire rods. They differ in that the sticks were thinner and lengths are measured in inches rather than centimeters.

Just as the squares of Gift Seven are the objects that surround and define a cube, so the sticks of the Gift Eight are the objects that surround and define a square. The two-dimensional, triangular and rectangular, tablets or tiles of the seventh gift involved a shift down from the three-dimensional rectangular solids of Gifts Three through Six. The eighth gift involves yet another shift down in dimension to sticks, which represent one-dimensional line segments. Froebel viewed dimensionality as an important idea and

took care in creating this progression of gifts so as to engage children in activities which provide the opportunity for them to experience and think about the differences between objects that are three-, two-, or one-dimensional.

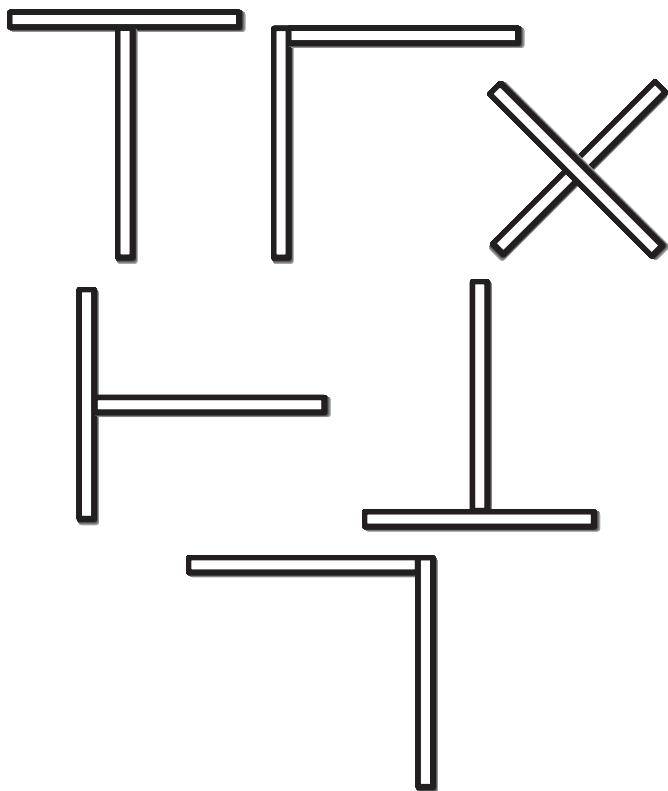
Always concerned that children not be given too many things at a time, Wiebe suggested that when introducing this gift each child should be given just one stick (probably one of the longer ones). The following is a possible dialogue between teacher and student suggested by Wiebe.

The child, holding the stick in his hand is asked: What do you hold in your hand? How do you hold it? Vertically. Can you hold it in any other way? Yes! I can hold it horizontally. Still in another way? Slanting from left above, to right below, or from right above to left below. Lay your stick upon the table. How does it lie? In what other direction can you place it? (Wiebe, p. 169)

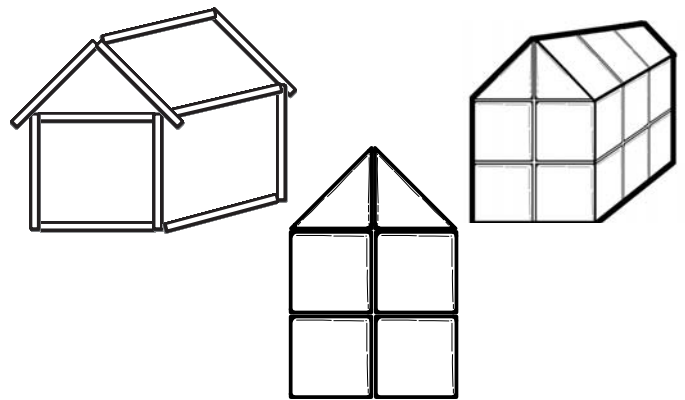


Whether or not the language used by Wiebe in this dialogue is realistic or appropriate for primary children, it is clear that the intent of this gift is the representation and exploration of lines and line segments.

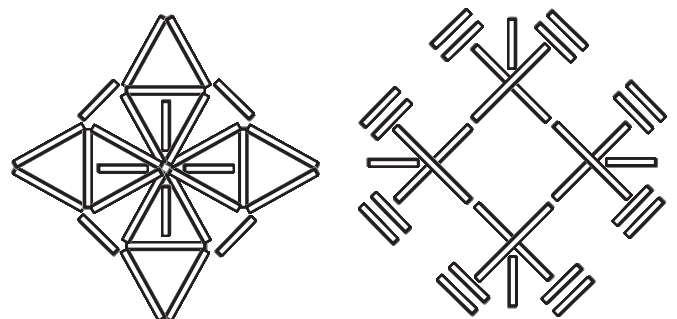
When children are subsequently given a second stick, they are asked about the various ways in which the two sticks can be put together: to form a T shape, an L shape, or a cross. Moreover, he asked them about the various ways in which these shapes might be oriented. For example, the L shape might be rotated 90 degrees or the T turned upside down or placed on its side. He goes on to have children form angles with the sticks and to remind them of the acute, obtuse, and right angles of the tiles that they explored and learned about in Gift Seven. With the move from one to two sticks, the children are using the sticks to form things that are two-dimensional and are asked to relate what they are doing with the sticks to the tiles of the previous gift. For example, with the seventh gift children identified triangles as acute, obtuse, or right and identified the angles of triangles as acute, obtuse, or right.



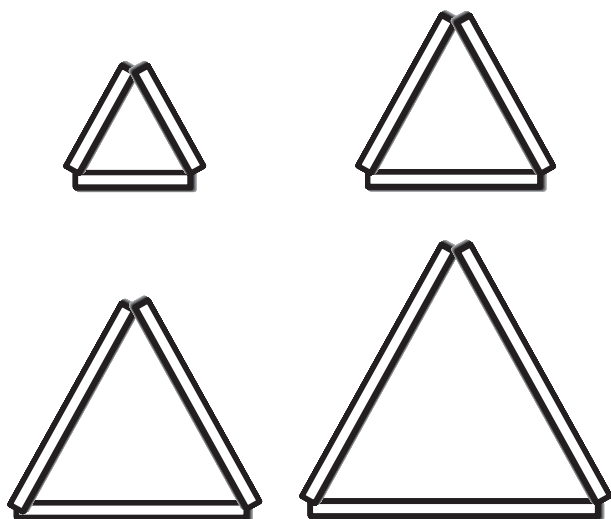
With the addition of more sticks, Wiebe suggests that children be asked to make representations of a variety of objects in their surroundings. For example, they might create a representation of a house, which they will already have done with the cubes and half-cubes of the earlier gifts and the squares and triangles of the seventh gift. The stick house looks more like the one created using the cubes and half-cubes than did the one made of the squares and triangles of the seventh gift, where only one face of the house is represented. The stick house represents in two dimensions what the house built of cubes and half-cubes represented in three dimensions. What is especially interesting about the representations made with the sticks is how much they are like what a child might create using paper and pencil. The sticks turn out to look very much like a drawing in which only line segments are permitted.



As he did with each of the other gifts, Wiebe goes on to have children create symmetric patterns using the various lengths and colors. The following are two examples of such arrangements.



Connecting back to the triangles of the seventh gift, children can be asked to create a variety of triangles and quadrilaterals and then describe how they are alike and how they are different. For example, they can construct five or six different equilateral triangles, one for each of the lengths of the sticks. By putting sticks end-to-end even larger triangles can be formed.



Shifting away from geometric ideas, Wiebe described ways to use the various lengths of sticks to represent numbers and number relations. For example, children are asked questions like, which stick has the same length as sticks of lengths 2 and 3 placed end-to-end; or which stick has the same length as three sticks of length 2 placed end-to-end?



It has been my purpose through this series of articles to give a description and use of each geometric gift as intended by Froebel. The sources have been writers like Wiebe who were passionate proponents of Froebel's work and attempted in their writing to faithfully interpret his work for subsequent generations of kindergarten teachers. I believe that these materials have a place in the kindergarten of the twenty-first century and it is my hope that over the next few years these gifts, along with supporting activities, will again be available to the children and teachers of kindergarten.

## References and Related Reading

Bosterman, Norman. *Inventing Kindergarten*. Harry N. Abrams, Inc. New York. 1997.

Froebel, Friedrich. *Pedagogics of the Kindergarten*. D. Appleton and Company. New York. 1909.

Fulmer, Grace. *The Use of the Kindergarten Gifts*. Houghton Mifflin Company. Boston. 1918.

Wiebr, Edward. *Paradise of Childhood*. Milton Bradley Company. Springfield, MA. 1923.

Wiggins, Kate and Nora Archibald Smith. *Froebel's Gifts*. Houghton Mifflin Company. Boston. 1986.