California State
AIMS activities supporting Sixth Grade Mathematics Standards of Learning

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

NUMBER SENSE

1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

"Fraction Actions 1-8, 15-19," Actions with Fractions
Students will use pattern blocks to determine equivalent fractions and order fractions from least to greatest.
"Parade of Triplets," Proportional Reasoning
Students will convert decimals to percents and will graph to find its fraction equivalent.
"Fraction Dominos/Rational Dominos," Proportional Reasoning
Students will play solitaire and group games using fraction dominos to determining equivalence or ordering fractions.

1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, a to b, a:b).

"Rolling Rectangles," Gravity Rules!
Students will determine how the aspect ratio effects the rate of fall by finding the aspect ratio of various size rectangles representing airplane wings and converting his ratio to the decimal equivalent.
"Glide Ratio," Gravity Rules!
Students will determine the glide ratio of a ram-air parachute. Use the video from Gravity Rules!
"Happy Landings" *AIMS*: III.7
Students will determine the efficiency of paper gliders by finding their lift to drag ratio.
"Tailor Made," Proportional Reasoning

"It Floats! It Sinks!" Floaters and Sinkers
Students find the density of irregularly shaped objects, by comparing their mass to volume ratio. Students observe how density effects the objects ability to float or sink in water.

1.3 Use proportions to solve problems (e.g., determine the value of N if 4/7 = N/21, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

"Rectangle Ratios," Proportional Reasoning
Students will find ways to prove that two rectangles are similar, by comparing the ratio of width to length and graphing those ratios on the coordinate plane.

"Proportional Practice," Proportional Reasoning
Students will develop a meaningful concept of fractions, ratios, percents, and proportions by using geoboard, grid, and segmented line models.

"Shadow Knows," Proportional Reasoning
Students will measure shadow lengths cast by objects of various heights to establish the understanding that the length of a shadow and the height of the object have the same proportion for all objects at any given time.

"Paring It Down to Size," Through the Eyes of Explorers
Students construct similar proportional characters and arrange them on a meter stick at intervals which cause them to appear as one. Taking measurements, from one shape to the other the students will be able to develop a proportion to compare size.

"Tunnel Vision," Through the Eyes of Explorers
Students use toilet paper tubes as range finders to discover that the ratio of the view's height to the distance from the eye to the object being viewed is proportional to the tube's diameter and length.

"Mirror Ricochet," Proportional Reasoning
Students will discover a method by which they can use mirrors, placed on the floor or sidewalk, to find the height of objects.

Resource: "Building Bridges to Algebra and Beyond," *AIMS*: XIII.9

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

"Percent Bands," Proportional Reasoning
Students will stretch the band to the length of an object. Using the predetermined marks, they will determine what percent of the object's length other objects are.

"Shrink Art," Proportional Reasoning
After heating the plastic, the students will compare the sizes of the pictures before and after to see if the shrinking was consistent in all directions.

"Proportional Practice," Proportional Reasoning
Students will develop a meaningful concept of fractions, ratios, percents, and proportions by using geoboard, grid, and segmented line models.

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

"Fraction Actions 32-53, 94-103," Actions with Fractions
Students will use pattern blocks to represent the addition of fractional parts.

"Fraction Actions 54-72," Actions with Fractions
Students will use pattern blocks to represent subtraction of fractional parts.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., 5/8 ÷ 15 /16 = 5⁄8 x 16 /15 = 2 /3).

"Fraction Actions 82-90," Actions with Fractions
Students will use the square model to represent multiplication of fractions.
"Multiplication of Fractions: What's Really Going On," *AIMS*: XIII.3

2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

"Finding Net Worth," *AIMS*: X.8
Students will use arrangements of integers on a number line to represent debts and assets to conceptualize addition of positive and negative integers.
"Patterns in Products," *AIMS*: X.9
Students will use patterns and analogies to construct their understanding of the multiplication of integers.
"Weight in Space," Out of this World
Students will determine their weight on the moon and other planets multiplying by the proportional surface gravity.
"Extra Terrestrial Excursions," Out of this World
Students will predict the amount of time it will take to get to the moon and other planets.
"Space Talk," Out of this World
Students will compute the time it would take for a series of messages to go back and forth between the moon and the planets.

2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

"Designer Gears," Machine Shop
Students will discover several methods of predicting how a design will look before drawing it by using the number of teeth on the gears. Students discover that the number of teeth used to complete the design is the least common denominator for the ring gear and design gear. Students also use a Venn diagram to find the least common denominator and greatest common divisor.
"Turn Around," Brick Layers
Students will pair different gear combinations to discover the inverse relationship of gear size to rate of rotation. Students will learn to use the least common denominator to solve this enigma.
"Greatest Common Divisor Meets the Least Common Multiple," Historical Connections Vol. 2
Students will discover the relationship between any two positive integers and their GCD and LCM.

"Euclid's Algorithm," Historical Connections Vol. II
Student's will use Euclid's Algorithm to find the greatest common divisor.

ALGEBRA AND FUNCTIONS

1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

"Recognizing and Building Proportional Relationships," Proportional Reasoning
Using ordered pairs, students will discover the difference between a proportional linear graph and a non-proportional linear graph.
"Wheeling Your Way to the Top," Brick Layers
Students will construct a winch using three different wheel sizes. They will crank the winch and measure the amount of string used by each wheel in a rotation to establish the relationship of wheel

size to rate of lift. Students will write a linear equation that tells how far from the floor a load will be if you turn the crank a given number of turns, and discover the slope line intercept form from the graph.

1.1 Write and solve one-step linear equations in one variable.

1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.

"Nuts and Bolts," Machine Shop
Students will determine the mechanical advantage for different nuts and bolts by measuring how far the nut moves with each rotation. Students will graph and write a linear equation for each bolt to predict the separation, knowing the number of rotations.

1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

"Four Fours," Historical Connections Vol. 1
Using exactly four fours and using the order of operations the students will make equations that equal 0,1,2,3,4,5,6,7,8, and 9.
"Building Blocks," *AIMS*: VIII.7
Using the Cartesian coordinate plane and base ten tiles, the students will discover a graphical representation of the distributive property.

1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator.

2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

"Paper Clip Chains," Proportional Reasoning
Students will measure objects with standard and jumbo paper clips. They will then graph and analyze the data to develop their understanding of proportional reasoning as related to rates and graphic displays.
"Mixing Measures," Proportional Reasoning
Students will measure a number of items in centimeters and inches to determine a conversion rate. They will make a scatter plot of the data to see the proportional relationship.

2.2 Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity.

"Time Trials " Proportional Reasoning
Students will measure the speed of a battery-powered vehicle to develop an understanding of speed.
"The Race," Gravity Rules!
Students will collect, record, and graph, in three different ways, the distance-time data of a race between a tortoise and a hare. Students will understand why a distance-time graph is so constructed.

2.3 Solve problems involving rates, average speed, distance, and time.

"Traveling Times," *AIMS*: XIII.9

Students will plot 12 ordered pairs that are factors of 60 on a graph and then use the graph to help them answer questions about rate and time. Students will understand that a graph of multiplication is a curved line.

"Pulse Rates," Proportional Reasoning

Students will measure their pulse rates under several conditions to see the difference numerically, graphically, and symbolically.

"Reading Rates," Proportional Reasoning

Students will explore the proportional meaning of rates by determining their reading rates and using them to determine how long it would take them to read a book.

3.0 Students investigate geometric patterns and describe them algebraically:

3.1 Use variables in expressions describing geometric quantities (e.g., P = 2w + 2l, A = 1 /2 bh, C = πd—the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

3.2 Express in symbolic form simple relationships arising from geometry.

"Practically Pi," Math + Science, a Solution

Students will understand that Pi is a constant relationship between the circumference and diameter of any given circle by measuring the circumference and diameters of various circles.

"Can You Believe It?" *AIMS*: II.8

Students will predict and compare the relationships between the circumferences and heights of various cans.

MEASUREMENT AND GEOMETRY

1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

1.1 Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.

"Practically Pi," Math + Science, a Solution

Students will understand that Pi is a constant relationship between the circumference and diameter of any given circle by measuring the circumference and diameters of various circles.

Resource: "Thinking About Circumference and Diameter," *AIMS*: IV.9

1.2 Know common estimates of π (3.14; 22 /7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base x height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

"Tin Can Space," Floaters and Sinkers

Students will estimate and calculate using the area of the base times the height and measure the volume of six different cans.

"How Much Cargo Will It Hold?" Floaters and Sinkers

Students will compare mass, volume, and density measures to determine which should be used for making fair charges for hauling cargo.

"Massive Boxes," Floaters and Sinkers

Students will fine the mass of various objects and note that mass is not necessarily related to the size (volume) and shape of an object.

2.0 Students identify and describe the properties of two-dimensional figures:

2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

"A Matter of Degrees," What's Next Vol. 2

Students will complete a table to discover a formula which gives the sum of the angles for any polygon with n sides.

2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

"Back Talk," *AIMS*: X.9

Students will determine what animal or geometric figure is on their back by asking their peers yes/no questions. They will then classify the animals or geometric figures and decide upon a strategy for determining what label they are wearing by asking the fewest questions.

STATISTICS, DATA ANALYSIS, AND PROBABILITY

1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

"The Marbleous Rolls," *AIMS*: VIII.1

Students will study the effect of uniform acceleration of marbles rolled down an inclined plane on the distance they roll on a carpet. Students will graph the data representing the range, mean, median and mode of the results.

"Penny Sort and Nickel Dates," Math + Science, a Solution

Students will classify pennies and nickels by minting dates; determine medians and modes; construct real, representational, and abstract bar graphs; and interpret results.

1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

"Stars in the Milky Way," Out of this World
Students will discover the method by which scientists estimate the number of stars n the Milky Way Galaxy by calculating the number of characters on a page in the classified section of the newspaper using a random sampling technique.

"Census Takers," Critters
Students will take samples of a critter population and estimate the total population from the samples.

"How Many Teddy Bears in the Woods," *AIMS*: IV.5
Large populations of highly mobile individuals are frequently estimated by the capture-mark-release-recapture method which is simulated in this activity.

"Disease X: The Crisis," *AIMS*: IV.10
In a simulation, students will compare blood profiles obtained by random sampling, and will make diagnoses by comparing these to predetermined profiles from the general population.

"Disease X: The Dilemma," *AIMS*: V.1
In a simulation, students will compare blood profiles obtained by random sampling, and will make diagnoses by comparing these to predetermined profiles from the general population.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

refer to 2.1

2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

refer to 2.1

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

refer to 2.1

3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:

Resources:
"Probability As Measurement," *AIMS*: VII.1
"Probability: Back to Basics," *AIMS*: VII.2
"New Probability from Old: Part II," *AIMS*: VII.4
"New Probability from Old: Part III," *AIMS*: VII.5

"An Area Model for Solving Probability Problems," *AIMS*: VIII.2
"Great Expectations on the Midway," *AIMS*: VIII.3
"Great Expectations on the Midway: Part III," *AIMS*: VIII.5

3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

"Toss for Triangles," What's Next ? Vol. 2
Students determine the probability that a triangle can be constructed from three line segments whose lengths are determined by tossing three dice.
"Scissors, Rock, or Paper," *AIMS*: III.5
Students will explore the probability of an event occurring by playing a game called Scissors, Rock, or Paper? They will determine both experimental and theoretical probabilities.

"Sum Domino Discoveries," *AIMS*: III.10
Students will find the experimental and mathematical probabilities of drawing a domino from a double-six set that has a prime number of dots.
"Teddy Bears Fight Pollution," *AIMS*: II.3
Students will discover that the probability of survival of an animal population is directly proportional to the amount of pollution.
"See How They Roll," Pieces and Patterns
Students will explore the mathematical and experimental probability of rolling three dice so that a given type of triangle will result.

3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

refer to 3.1

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1-P is the probability of an event not occurring.

refer to 3.1

3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5 Understand the difference between independent and dependent events.

California State

AIMS activities supporting Seventh Grade Mathematics Standards of Learning

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

NUMBER SENSE

1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:

1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

"Parade of Triplets," Proportional Reasoning
Students will convert decimals to percents and will graph to find its fraction equivalent.

"Oranges for the Most Part," *AIMS*: X.5
The student will convert fractions to decimals to percent while finding edible portions of oranges.

1.4 Differentiate between rational and irrational numbers.

"Decimal Predictions," What's Next Vol. 3
Students use a pattern to predict the 50th digit for each repeating decimal representation.

"From Fractions to Decimals," What's Next Vol. 2
Students write the first twenty decimal places for a series of fractions to determine a pattern for predicting the remaining digits.

1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

1.6 Calculate the percentage of increases and decreases of a quantity.

1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

Resource: "Encountering Percents," *AIMS*: XII.8

2.0 Students use exponents, powers, and roots and use exponents in working with fractions:

2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

2.2 Add and subtract fractions by using factoring to find common denominators.

2.3 Multiply, divide, and simplify rational numbers by using exponent rules.

2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

"Heron's Square Root Method," Historical Connections Vol. 2
Students use Heron's square root method for finding square roots.
Resource: "Developing the Concept of Square Root: A New View," *AIMS*: III.1

2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

ALGEBRA AND FUNCTIONS

1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:

1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

"Collecting Terms and Constructing Equations," *AIMS*: XIII.3
Students are introduced to the concept of variables and collecting terms through the use of manipulatives and graphic representations.
"Building Picket Fences," *AIMS*: XIII.1
Students will study pictures of picket fences to determine patterns in the number of pickets and nails added, and writing their findings as a general expression.

1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

"Four Fours," Historical Connections Vol. 1
Using exactly four fours and using the order of operations the students will make equations that equal 0,1,2,3,4,5,6,7,8, and 9.

1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

"Collecting Terms and Constructing Equations," *AIMS*: XIII.3
Students are introduced to the concept of variables and collecting terms through the use of manipulatives and graphic representations.

1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

"What's My Line?" *AIMS*: X.5
Students will learn to interpret the line of a graph by relating their hands-on manipulation of volume and height of water in a bottle to the line that is produced.
"Fall-timeters," Gravity Rules!
Students will collect and record time and altitude data directly from the video of a skydiver. They will then graph and analyze the data and use it to compute average velocities, including terminal velocities.
Resource: "Four Families of Relationships," *AIMS*: XIII.10

2.0 Students interpret and evaluate expressions involving integer powers and simple roots:

2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

"Calculator Fun," Historical Connections, Vol. II
Students will perform computations on a calculator and read the resulting word when the calculator is turned upside down.

2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

3.0 Students graph and interpret linear and some nonlinear functions:

3.1 Graph functions of the form y = nx 2 and y = nx 3 and use in solving problems.

"Constant Areas," *AIMS*: XIII.7
Students will explore the concept of area as they look for patterns on a multiplication chart and begin to create their own multiplication chart by graphing rectangles that have the same area.

"Welcome to 42nd Street," *AIMS*: XIII.8
Students will graph ordered pairs (x, y) whose product is 42 in the first quadrant. They will then construct a three-dimensional model of the structures on 42nd Street to provide an additional perspective from which to view the results.
"Base Jumping," Gravity Rules!

Students will discover how BASE jumpers determine how long they can safely freefall before opening the parachute by: 1. Graphing and interpreting the distance-time data contained in a BASE Jumper Table. 2. Collecting delay time data from a video segment showing BASE jumpers jumping off the New River Gorge Bridge. Students then will use delay times to compute altitudes.

"A Swing in Time," *AIMS*: XI.4

Students will study the variables affecting the motion of a pendulum and discover that the length of the pendulum is the variable that effects the rate of the pendulum's swing. They will gather and graph data about different pendulum lengths to determine the predictability of the period. The graph produces a parabola.

"Threads of Time," *AIMS*: XI.4

Students will make pendulums with cycles of 1-, 2-,3-, and 4-seconds. Using their data they will construct a graph and determine an equation relating length of the pendulum to its period.

3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

3.3 Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

"Patterns in Equivalent Fractions," Proportional Reasoning

Students graph fractions in the coordinate plane and discover how the slope of the line is related to the magnitude of the fraction.

3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

"Around and Across," Proportional Reasoning

Students graph the diameters and circumferences of a variety of circles to find that the slope of the line of best fit is "pi" 3.14.

"Sizing Up Shadows," Through the Eyes of Explorers

Students will measure and graph shadow lengths cast by dowels of various heights to establish the understanding that the length of a shadow and the height of the object have the same proportion of all objects at any given time.

"Drop It!" Proportional Reasoning

Students will compare the height of a ball's bounce from different drop heights. They will collect , graph, and analyze the data to determine a bounce to drop ratio which is converted to a unit ratio and to the decimal and percent equivalents.

4.0 Students solve simple linear equations and inequalities over the rational numbers:

4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

"Algebra Magic," Historical Connections Vol. III

Students will write a two step linear equation to explain why a magic trick works.

"Sandbagging the Seesaw," Machine Shop

Students explore the properties of effort, resistance, and torque using a seesaw as a first-class lever and write equations to find an unknown.

4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

"Time Trials," Proportional Reasoning
Students measure the speed of a battery-powered vehicle to develop an understanding of speed.
"Making Good Lessons Better," Proportional Reasoning

MEASUREMENT AND GEOMETRY

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

"Squaring Up Circles," Proportional Reasoning

"How Fast Can You Walk," Gravity Rules!
Students will measure the time interval it takes them to walk, at their fastest rate, through a known distance. They will use this data to compute their fastest walking velocity in feet per second, and then in miles per hour.
"Playing at Math," Proportional Reasoning
Students will measure scaled toy cars and use scaling techniques to determine the actual dimensions of a real car. They will use the scale factor to determine the scaled speed of their toy in feet per second and then in miles per hour.

1.2 Construct and read drawings and models made to scale.

"Its a Court Case," Through the Eyes of Explorers
Students measure a basketball court and make a scale drawing of it.
"Honey I Shrunk the ...," Through the Eyes of Explorers
Students will draw a map of the classroom or another room to the scale of a toy action figure.
"Target Practice," Through the Eyes of Explorers
Students will use a simple transit to measure the degrees from two different places on a baseline to an object in a field. By using similar triangles and a scale drawing, they will determine unknown distances.
"I Spy," Proportional Reasoning
Using an aerial photograph of Washington, DC and some information about the landmarks it contains, students will develop a scale, determine the distance between two locations, and estimate the time it would take to walk between the locations.

1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

"Flags of the Elements on Parade," *AIMS*: XIII.10
Students determine either the volume or the density of 19 different elements from information provided in the table with a constant mass of 60 grams. When students have determined all of the

density and volume values they will create two- and three-dimensional graphs of their results. The resulting graph is a curve representing the function 60 = xy

"Time Trials," Proportional Reasoning

Students measure the speed of a battery-powered vehicle to develop an understanding of speed.

"Making Good Lessons Better," Proportional Reasoning

2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:

2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

"Pulling Strings," Soap Films and Bubbles

Students will discover ,using soap film what shape provides the greatest area for a given perimeter with examination of area formulas.

Resources:

"Area Formulas on the Geoboard," *AIMS*: III.3

"Tangrams and Area," *AIMS*: III.9

2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

Spatial Visualization

Fifty-three activities in which students use rectangular solids to build structures from a given plan and then determine perimeters, surface area, volume, vertices, isometric drawings of the structure.

"Playing at Math," Proportional Reasoning

Students will measure scaled toy cars and use scaling techniques to determine the actual dimensions of a real car. They will use the scale factor to determine the scaled speed of their toy in feet per second and then in miles per hour.

"Color Tiles," XII.9

Students will extend patterns of different colored tiles as they grow sequentially. By counting and recording the quantities of each color of tile, they will recognize numeric patterns which they will represent graphically and symbolically. This experience will help students see the integration of a context with the graphic and symbolic representation.

"Painted Cubes," XII.10

Students will be able to determine the quantity of cubes painted on 0, 1, 2, and 3 sides in a large cube made of many small cubes. Students will determine the perimeters and surface areas of the faces.

2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft 2] = [144 in 2], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in 3] = [16.38 cm 3]).

3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:

3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

"Exploring the Amazing Circle 1-12," The Amazing Circle
Twelve activities in which students discover geometric properties of the circle through folding, creasing, and observing a paper circle.

3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

"Growing Designs," Proportional Reasoning
Students will enlarge designs on graph paper and then will study the coordinates and dimensions of the enlargements to gain an understanding of scaling.

3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

"A Pythagorean Puzzle," Historical Connections Vol. 1
Students work on a puzzle in which puzzle pieces form squares on sides a and b of a right triangle. The object is to combine those pieces to form a large square on side c showing the Pythagorean theorem.
"Probably Pythagorean," Pieces and Patterns
Students will determine what the probability will be that the resulting triangles will be acute, obtuse, or right triangles by applying the Pythagorean theorem.
Resources:
"The Pythagorean Relationship - Part 1," *AIMS*: V.2
"The Pythagorean Relationship - Part 2," *AIMS*: V.3

3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

"Boxes, Boxes, Boxes," *AIMS*: XI.7
Students will construct all possible nets to determine the best way of packaging 12 one inch cubes.
"Box It Up," Historical Connections Vol. 3
Students will hinge six squares together in various arrays to form a box, and draw all possible ways of doing so.

3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

STATISTICS, DATA ANALYSIS, AND PROBABILITY

1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:

1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

"Is Anyone Normal?" Proportional Reasoning

Students will gain understanding of measures of central tendency and distribution by using height data from the class to generate stem and leaf plots and box and whisker plots.

"Drops on a Penny Revisited" *AIMS*: XII.5

Students will gather data to draw conclusions as to how many drops of water can fit on a penny. They will study the central tendency and dispersion of the data to determine what variables affect the data most. By making this study students will gain an understanding of surface tension and what variables affect its strength.

"Color Samples," *AIMS*: XII.8

Students will gather the data for eight to ten bags of M&M candies, make graphic displays, and determine measures of central tendency for the samples. They will summarize their findings to make predictions of other bags.

1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

"Head Hunters," Proportional Reasoning

Students determine if there is a correlation between a person's height to head circumference. Students take measurements of the class members and analyze the data with a scatter plot and averaging.

"This is so Typical," Proportional Reasoning

Students will gather and analyze data to determine some of the physical proportions of a typical student.

1.3 Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Refer to 1.1

California State
AIMS Activities supporting Eighth Grade Mathematics Standards of Learning

ALGEBRA I

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:

1.1 Students use properties of numbers to demonstrate whether assertions are true or false.

2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

"Earthquake Mathematics," Historical Connections Vol. I
Students use exponential rules in dividing exponents to determine how much stronger a given earthquake magnitude is to another.

3.0 Students solve equations and inequalities involving absolute values.

4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.

"Collecting Terms and Constructing Equations," *AIMS*: XIII.3
Students are introduced to the concept of variables and collecting terms through the use of manipulatives and graphic representations in the first quadrant of the Cartesian coordinate plane.
"Collecting Terms II," *AIMS*: XIII.4
Students are introduced to negative terms and representation of those terms in the 4th quadrant of the Cartesian coordinate plane.

5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

"Algebra Magic," Historical Connections Vol. III
Students will write a two step linear equation to explain why a magic trick works.
"Algebra Solves the Mystery," Historical Connections Vol. III
Students will use algebra to explain the mystery in a card trick.

6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

"Nuts and Bolts," Machine Shop
Students will determine the mechanical advantage for different nuts and bolts by measuring how far the nut moves with each rotation. Students will graph and write a linear equation for each bolt to predict the separation, knowing the number of rotations.

"Wheel Your Way to the Top," Brick Layer

Students will construct a winch using three different wheel sizes. They will crank the winch and measure the amount of string used by each wheel in a rotation to establish the relationship of wheel size to rate of lift. Students will write a linear equation that tells how far from the floor a load will be if you turn the crank a given number of turns, and discover the slope line intercept form from the graph.

"A Shift in Lift," Brick Layer

Students will construct a winch with a winding wheel on each end. While keeping one wheel the same size, they will change the wheel size on the other end. Students will hang masses on each wheel to keep them in balance, and determine how the wheel size affects the force generated by the wheel. Using the data they gather, students will make broken-line graphs to visualize the information and help in constructing an equation.

7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

"Making Connections," *AIMS*: X.1

Students learn to multiply binomials using tiles placed in the first quadrant of the Cartesian coordinate plane.

"Solid Experience," *AIMS*: X.4

Students use algebra-blocks to build a rectangular solid from a set of given dimensions, where n= 3. The resulting volume is related to multiplying polynomials. Numerical and algebraic equations are written and isometric drawings made to increase this awareness.

"Another Solid Experience," *AIMS*: X.5

Students use algebra-blocks to build a rectangular solid from a set of given dimensions, where n = 4. The resulting volume is related to multiplying polynomials. Numerical and algebraic equations are written and isometric drawings made to increase this awareness.

"Putting It All Together," *AIMS*: X.7

Students will construct rectangular parallelepipeds using the components listed for each activity. Each set of components will build a unique box-like figure since any third degree equation has one and only one set of factors.

"A New Model for Quadratic Equations," *AIMS*: X.10

Students will use the first and fourth quadrant of the coordinate plane and algebra tiles to build rectangles with a given set of components , placing negative components in the fourth quadrant.

"The Binomial Theorem," What's Next? Vol. 3

Students use Pascal's Triangle to find the pattern for expanding a binomial without doing a lot of tedious work.

11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

"Making More Connections," *AIMS*: X.2

Students make a rectangular array from a given set of components where n=4, the area resulting in a polynomial. By finding the length and width dimensions the polynomial is factored.

"Exploring Rectangles," *AIMS*: X.3

Students fill in a given rectangular area with algebra tiles learning that factoring polynomials is the same as finding the length /width dimensions of the rectangle.

"From Pictures to Equations," *AIMS*: X.6

Students write numerical and algebraic dimensions of an isometric drawing of a rectangular parallelepipeds as a means of factoring third degree expressions.

"Number Bases," *AIMS*: XII.9

Students write the equation for width x length = area in terms of base 4 using exponential notation for various rectangular arrays.

12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

14.0 Students solve a quadratic equation by factoring or completing the square.

15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

19.0 Students know the quadratic formula and are familiar with its proof by completing the square.

20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

21.0 Students graph quadratic functions and know that their roots are the x-intercepts.

"Base Jumping," Gravity Rules!

Students will discover how BASE jumpers determine how long they can safely freefall before opening the parachute by: 1. Graphing and interpreting the distance-time data contained in a BASE Jumper Table. 2. Collecting delay time data from a video segment showing BASE jumpers jumping off the New River Gorge Bridge. Students then will use delay times to compute altitudes.

"A Swing in Time," *AIMS*: XI.4

Students will study the variables affecting the motion of a pendulum and discover that the length of the pendulum is the variable that effects the rate of the pendulum's swing. They will gather and graph

data about different pendulum lengths to determine the predictability of the period. The graph produces a parabola.

"Threads of Time," *AIMS*: XI.4

Students will make pendulums with cycles of 1-, 2-,3-, and 4-seconds. Using their data they will construct a graph and determine an equation relating length of the pendulum to its period.

22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.

23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

24.0 Students use and know simple aspects of a logical argument:

24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.

24.2 Students identify the hypothesis and conclusion in logical deduction.

24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:

25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.

25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.

25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.