

BRIDGE CROSSING CHALLENGE

Puzzle Topic
Network Theory

Puzzle Question
In each diagram, how can you cross every bridge once, and only once?

Materials
Student sheets

Puzzle Background

In the town of Königsberg, Prussia (now Kaliningrad, Russia), there were seven bridges over the Pregel river connecting two islands and various other parts of the city. There was a tradition in Königsberg to try and walk around the city, crossing over each of the seven bridges only once. No one was ever able to do it, and yet they were unable to prove that it could not be done. In 1735, some college students sent the problem to Leonhard Euler (1707-1783), one of the greatest mathematicians of the day. Euler was able to mathematically prove that this walk is impossible because of the number of points that were connected by an odd number of paths. Euler's study of topics like this earned him the title, *The Grandfather of Topology* because he unknowingly studied the topics that would come to be an integral part of that discipline when it was introduced over 100 years after his death.

This activity challenges students to solve some puzzles similar to the Bridges of Königsberg problem. The original problem states: *Königsberg is a city on the Pregel River that contains two islands and is joined by seven bridges. The river flows around the two islands of the town. The bridges run from the banks of the river to the two islands in the river with a bridge connecting the islands. It became town tradition to take a Sunday walk to try and cross each of the seven bridges only once. Draw the map the people must have followed in order to complete their journey.*

Each puzzle in *Bridge Crossing Challenge* asks students to draw a route that will cross every bridge once and only once. After students have solved each puzzle (or determined that it is not possible to solve), they will create a bridge crossing puzzle of their own to share with a classmate. This puzzle should help students see some of the more real-life applications of the study of topology and networks.

Puzzle Presentation

1. Each student will need a copy of the student sheets for this activity. Be sure that students use pencils so that they can make mistakes and try several different methods, if necessary.
2. Not all of the puzzles in this activity can be solved; however, students should be allowed to discover this fact for themselves.
3. When students create their own puzzles to share with a classmate, you may want to have them draw the final copy in pen so that the other students can erase pencil marks without erasing the puzzle.
4. If you would like to share some of the history of this kind of puzzle with your students, a copy of the Bridges of Königsberg has been included along with a very basic description of Euler's proof.

Solution Hint

Look at the number of bridges that go to each land mass.

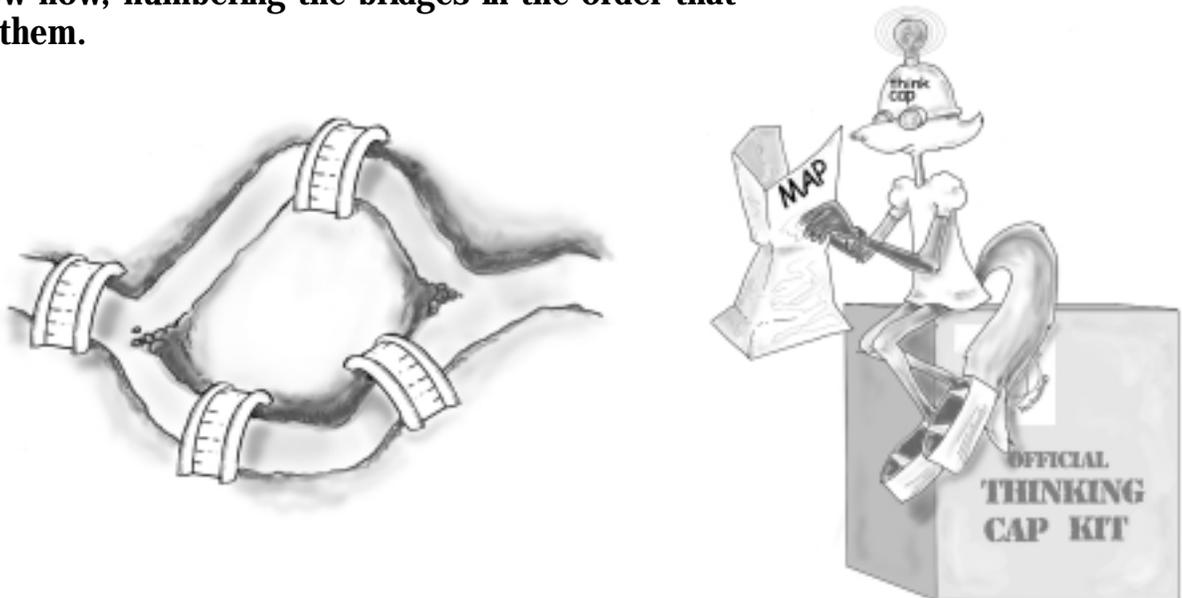
BRIDGE CROSSING CHALLENGE

Read each scenario below and follow the directions.

There is an island in the center of Elizabeth's town that has three bridges leading to it. Elizabeth likes to take a walk from her home to the island by crossing every bridge once and only once. How can she do this? Draw Elizabeth's home where it belongs.

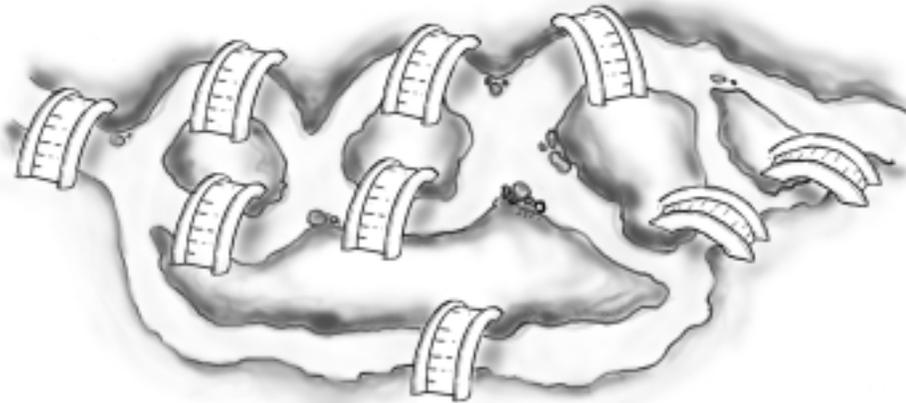


Carlos is trying to find a way to cross every bridge on the river in his town without ending up on the island. Can he do it without crossing any of the bridges more than once? Show how, numbering the bridges in the order that you cross them.

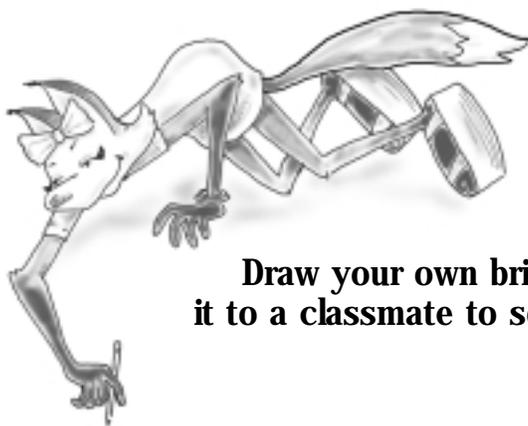


BRIDGE CROSSING CHALLENGE

The Spiral River in Lin's town has nine bridges that connect five islands. Every year the Five-Island race goes through the town crossing each bridge only once. Show one route that the racers could take, numbering the bridges in the order they should be crossed.



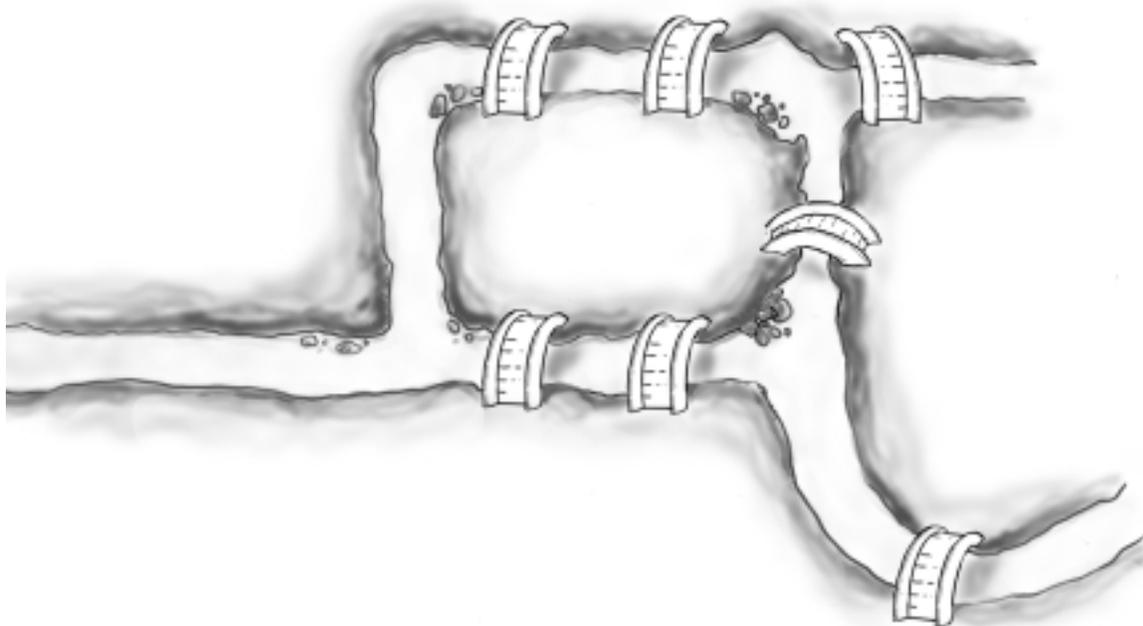
The Revolution River creates two nearly identical islands that are each accessible by three bridges. The Euler Foundation, a new company in town, is offering \$20,000 to anyone who can visit both islands by crossing each bridge once, and only once. Can you win the prize? Show how.



Draw your own bridge crossing problem in the space below and give it to a classmate to solve.

THE BRIDGES OF KÖNIGSBERG

Königsberg is a city on the Pregel River that contains two islands and is joined by seven bridges. The river flows around the two islands of the town. The bridges run from the banks of the river to the two islands in the river, with a bridge connecting the islands. It became town tradition to take a Sunday walk to try and cross each of the seven bridges only once. Draw the map the people must have followed in order to complete their journey.



This is the original bridge crossing problem that a man named Leonhard Euler solved in the 1730s. He proved that it was impossible to cross each of the seven bridges in a single walk without retracing some of your steps. In fact, when dealing with bridges, if there are three or more pieces of land that have an odd number of bridges going to them, you will not be able to cross each bridge without retracing some steps.

How does this knowledge compare to what you discovered as you solved the puzzles?

Draw two bridge-crossing puzzles on the back of this page, one that is possible, and one that is not. Give them to a classmate and have the classmate determine which can be solved and which cannot without actually doing either one.